1. Leveling and Mapping

Accurate maps of the daily pressure and current fields were obtained by combining the leveled pressure measurements and current measurements in a multivariate, nondivergent OI procedure. Maps of the streamfunction field p could have been generated from $[u, v]$ current measurements alone. The resulting maps would have been spatially consistent since the current measurements were spaced close enough to be well correlated. However, each daily map would have had a different, undetermined reference level. In other words, they would be temporally inconsistent by an amount comparable to the eddy signals. This is physically reasonable, since p maps generated (as streamfunctions) solely from velocity measurements stem from only the measured *g*radients of pressure. On the other hand, the dedrifted pressure P' records from bottom pressure sensors are temporally consistent. Yet by themselves, they are not suitable inputs for OI mapping until they have been leveled, because each site has an arbitrary level (or reference pressure) relative to the others, *i.e.*, they would be spatially inconsistent. Once leveled, the pressures alone could have been used to produce spatially and temporally consistent maps. However, by combining them with the measured currents in a multivariate mapping procedure more accurate mapped fields were produced. Reduced mapping errors were obtained because pressure as well as its gradients were supplied to the mapping procedure. Furthermore, by our choice of covariance functions, the output fields $[P, u, v]$ were constrained to be geostrophic (Bretherton et al. 1976).

a. *Leveling*

Leveling was accomplished by assuming that the measured *mean* abyssal currents were geostrophic and determining the reference pressures accordingly. Additional assumptions are implied in combining the dedrifted P, u, v data for leveling: (a) The dynamic pressure at the ocean bottom does not change from its geostrophic values (other than hydrostatically) just above the bottom boundary layer, an assumption which is well justified theoretically (Pedlosky 1987). (b) The vertical shear (thermal wind) of the currents is small at the 3500 m level, as is observed from the smallness of lateral gradients of density, so that horizontal pressure gradients measured near the bottom are the same as at 3500 m.

The leveling procedure is diagrammed at the top of the flowchart in Figure 2. The OI technique was applied on the current measurements alone to map the streamfunction (p) fields (the process is designated as "u2p" in the upper right corner). The interpolation of these streamfunction fields to the sites of the PIES instruments yielded a set of pressure time series, each with an offset that is only time dependent (spatially uniform). Let $p(t, s)$ and $p_0(t)$ denote at site s this optimally-interpolated pressure time series and its time-dependent unknown offset from the absolute pressure. Let $P'(t, s)$ and $P_0(s)$ represent at site s the measured (dedrifted) pressure and its site-dependent unknown reference level (temporally constant). Since the absolute pressure at a given time and site is unique,

$$P'(t, s) + P_0(s) = p(t, s) + p_0(t).$$

The unknown $p_0(t)$ may be eliminated by subtracting the above equation for any one site (we used site G2) from the equations for all other sites, giving

$$\Delta_s P_0(s) = \Delta_s p(t, s) - \Delta_s P'(t, s), \tag{1}$$

where Δ_s is the spatial difference operator. The left hand term is independent of time. However, the right hand side, estimated from real data, varies with time due to errors primarily arising from mapping $p(t, s)$ each day. Changes in $P(t, s)$ between 3500 m and the ocean bottom associated with non-zero vertical shear above each PIES may possibly contribute to the error as well. Therefore, for each site the reference level on the left hand side was calculated as the time average of the right hand side where the averaging period is the overlapping time of good pressure data between site s and site G2. If the measurement periods for all sites were identical the last term in Eqn (1) would average to zero, and the above procedure would simplify to applying nondivergent OI on the mean current records to estimate the mean streamfunction at the pressure measurement sites. These mean streamfunction values could then be used as the pressure reference levels[1].

In essence, the mean geostrophic pressure field determined from current records was used to produce spatially consistent reference pressures for the measured P'. The sum of a residual pressure record and its reference pressure, $P(t, s) = P'(t, s) + \Delta_s P_0(s)$, will be denoted as the leveled pressure record. The only remaining undetermined constant is independent of location and time, *i.e.* a dynamically unimportant constant.

b. *Multivariate OI Mapping*

The multivariate mapping step (denoted "pu2p" and "pu2u") is indicated midway down Figure 2 and the mathematical details are given in the Appendix. The resulting maps are termed "preliminary" because of a refinement, which is described next.

The OI analysis procedure operates with variables that all have zero mean. Consequently, it is best to first remove a mean field from the input data and subsequent to the OI, to restore a consistent mean to the output. The preliminary maps were generated assuming that the means were zero. Although the mean pressures differed by less than 0.08 dbar between sites, there was a discernable spatial distribution to the means. Accordingly, the 26-month-long temporal mean pressure field and its corresponding geostrophic velocity were calculated from the maps produced during the preliminary pass of the multivariate OI (Figure 2). Then, the means were subtracted from the leveled pressure and current records. Next, the OI procedure was applied to the "demeaned" records to produce residual fields. Lastly, the mean fields were restored to produce the final sequence of pressure and current maps. A result of improving the mean field specification is that the final maps exhibited less smoothing than the preliminary versions.

The preliminary OI step was used only for finding averages of currents or pressures. The results are not particularly sensitive to the choice of correlation length r_0 and the noise-to-signal ratio $\mathcal{E}_0$ for the OI mapping (defined in Appendix), which we chose for this step to be 150 km and 0.05, respectively. These long correlation and low noise values are appropriate for mapping the average fields. For the final OI step, in which the eddy residual pressure and current fields were mapped, the value for r_0 was chosen by least-squares fitting a Gaussian function to the spatial correlation function estimated from the residual data. The value for $\mathcal{E}_0$ was determined from the estimated uncertainty in the measured data and their residual variance (Qian and Watts 1992). We found

[1] Choosing an alternate reference than G2 would have produced a different constant offset for all the pressure maps, effectively a hydrostatic offset with no change in the dynamic current fields. Gappy data could have necessitated another approach to determine the best reference offsets from periods of overlapping good data. Fortunately our dataset had few data gaps, so the choice of reference site has no substantial effect upon our mapped fields.

$r_0 = 90$ km and $\mathcal{E}_0 = 0.10$, as are appropriate for the shorter correlation lengths and smaller signal levels in the residual eddy fields.

Mathematically, the OI procedure also assumes that the input data have spatially isotropic correlation functions with spatially uniform variance. Even though the observed variance was not uniform, the pressure and current data were not scaled by a variance field for two reasons. Firstly, nonuniformly-scaled pressure and current data would no longer be in geostrophic balance, resulting in much more complex cross-covariance functions between pressure and current. Secondly, the root-mean-square (rms) current speeds at different sites range only from 3 to 9 cm s^{-1}; thus scaling by the local variance should not be necessary. In any case, Watts et al. (1989) found that even in the upper layer jet, where nonuniform variance was more clearly evident, the mapped fields were not particularly sensitive to this prescaling.

The Appendix explains further mathematical details and presents the covariance functions which were fitted to the data. The OI procedure is fully documented in Qian and Watts (1992).

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