Chapter 5

OCEAN RESPONSE TO TROPICAL CYCLONE

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[This chapter benefited from careful review and material provided by Lynn K. Shay of the University of Miami]

5.1 INTRODUCTION

The substantial impact of sea-surface temperature (SST) on the genesis and intensification of tropical cyclones has long been recognized (Palmen 1948; Miller 1958). It is well known that tropical cyclones climatologically develop only over warm oceans with SSTs of 26°C or higher. Both theoretical (Emanuel 1986) and numerical models (e.g., Chang 1979; Tuleya and Kurihara 1982) also indicate a great sensitivity of maximum (or potential) storm intensity to SST. Emanuel (1988) developed a theory of the tropical cyclone intensity in which a storm is treated as a Carnot heat engine. In this theory, the maximum intensity increases rapidly for increasing SST. DeMaria and Kaplan (1994) derived an empirical relationship between climatological SST and the maximum intensity of Atlantic hurricanes based on a 31-year sample and found a similar sensitivity of tropical cyclone intensity to SST. Extensive review of observational and theoretical studies on the role of SST in the tropical cyclone evolution is even in Chapter 2.4.3.

Tropical cyclone modellers have primarily been concerned with perplexing problems of convective parameterization, vortex movement, and vortex-flow interaction (see Chapters 3 and 4). One potentially significant constraint on dynamical predictions of tropical cyclones is the lack of knowledge about the ocean response to the storm forcing. In almost all research and operational dynamical models, conditions of fixed SST in time are assumed, even though numerous observational and numerical studies have shown that tropical cyclones produce significant SST changes. SST may decrease up to 6°C as a result of strong wind forcing. Turbulent mixing in the upper oceanic layer accompanied by the mixed layer deepening and entrainment of cooler thermocline water is the primary mechanism for the SST decrease during the tropical cyclone passage (Price 1981; Black 1983; Bender et al. 1993). The heat fluxes to the atmosphere account for less than 20% of the total SST decrease.

Because of the sea-surface cooling associated with storm forcing, the tropical cyclone-ocean system is one with positive and negative feedbacks. During the genesis and development stages, a positive feedback in the tropical cyclone-ocean system exists. As the tropical cyclone strengthens, the evaporation rate grows due to the increase in the surface wind speed. The enhancement of the moisture supply from the ocean leads to an increase in the latent heat energy that drives the circulation of the tropical cyclone. As the storm continues to intensify, the increasing surface wind stress generates strong turbulent mixing that deepens the ocean mixed layer. The associated SST decrease may then result in a reduction of the total heat flux (latent plus sensible) into the atmosphere and lead to a decrease in storm intensity. This process represents a negative feedback mechanism.

The aim of this chapter is to present an overview of the main features of the ocean response to tropical cyclones based on observational and theoretical studies. Chapter 5.2 presents observational evidence of the ocean response to tropical cyclone forcing. Theoretical and numerical modeling studies to be discussed in Chapter 5.3 and Chapter 5.4 have helped to understand and evaluate various aspects of the ocean response to tropical cyclones. Although numerical ocean models have recently demonstrated realistic simulations of the ocean response, the potential negative feedback to the tropical cyclone remains poorly understood. Three-dimensional, coupled atmosphere-ocean models have recently been developed to simulate the mutual response of a tropical cyclone and the ocean. Chapter 5.5 contains a discussion of how the tropical cyclone-ocean interaction may
affect the surface heat flux, intensity, and the ocean. Further advances on this problem require comprehensive simulations with high resolution atmospheric and oceanic numerical models that can be tested against high-quality observations.

Another aspect of the ocean response to tropical cyclones is the generation of surface waves. Surface sea-state and its potential influence on the momentum and heat exchange between the ocean and a tropical cyclone are discussed in Chapter 5.6. Finally, one of the most destructive consequences of the ocean response to a tropical cyclone is the rapid rise in the sea-surface elevation near a coast called the storm surge. Chapter 5.7 describes some observations of storm surges and various empirical and numerical models developed for surge predictions.

5.2 OBSERVATIONS OF THE OCEAN RESPONSE TO TROPICAL CYCLONES

5.2.1 Thermohaline variations

For many years, the major source of data on the ocean thermodynamic response to tropical cyclones was the measurements collected from merchant ships. Although these data were irregular and incomplete, they clearly indicated that SST usually decreased by several degrees Celsius due to tropical cyclone forcing. Fisher (1958) documented a SST decrease of 3°C by analyzing merchant ship reports after the passage of tropical cyclones Connie and Diana in 1955. Jordan (1964) was the first to notice that the maximum SST decrease occurs on the right side of the track. He also pointed out that a major reason for the upper-ocean cooling is the vertical turbulent mixing induced by the strong wind stress associated with a tropical cyclone. Stevenson and Armstrong (1965) have found that tropical cyclones cause temperature changes not only at the sea surface but also in the ocean interior. They observed a temperature decrease of 2.5°C at depths greater than 80 m after the passage of Hurricane Carla (1961) in the Gulf of Mexico. Ogata (1960) and Maeda (1965), who analyzed vertical temperature profiles at ocean weather station Tango (29°N, 135°E), found that SST decrease during typhoon forcing was accompanied by deepening of the surface mixed layer by about 30 m. They also reported that the mixed-layer depth usually decreased about one day after the typhoon passage due to upwelling induced by the storm. Hazelworth (1968) analyzed the ship and buoy measurements of SST variations that resulted from 10 hurricanes and concluded that the SST reached minimum values about one day after storm passage and primarily on the right side of the track.

These early studies clearly showed that a better understanding of the ocean response to tropical cyclones would require systematic measurements of the three-dimensional thermodynamic ocean structure before, during, and after the storm passage. The first scientific experiment specifically designed for this purpose was conducted during research cruises in the Gulf of Mexico across the track of Hurricane Hilda in 1964 (Leipper 1967). Before the hurricane, the SST exceeded 29°C over almost the entire area of the Gulf. After the hurricane passage (with an average speed of 3-4 m s⁻¹ and maximum wind of 50 m s⁻¹), four crossings of the storm track were made with soundings to a depth of 270 m. The maximum SST decrease reached 6°C as a result of hurricane forcing. Strong upwelling along the storm track led to an almost complete disappearance of the surface mixed layer in that area, which contributed to the extreme cooling of the upper ocean.

Wright (1969) analyzed two cross-sections in the Kuroshio region less than one week before and two days after the Typhoon Shirley passage. This typhoon had a maximum wind of 40 m s⁻¹ and moved with an average speed of 13-15 m s⁻¹. The temperature decreased about 2°C near the surface and 3°C at depths of 80-170 m, with maximum decreases to the right of the storm track. Black (1972) observed a rather large SST decrease of 4°C due to the relatively weak Hurricane Ginger in 1972, which had maximum winds of only 35 m s⁻¹. However, the storm translation speed in the area was only 1 m s⁻¹, so the residence time of storm over the ocean was longer.
Fedorov (1972) analyzed 14 cases of tropical cyclone passage near ocean weather station Tango. He distinguished different types of vertical temperature anomalies (before and after a tropical cyclone) within the upper 200 m. The Type C profile is characterized by cooling throughout the water column, with the maximum cooling near 30 m depth. Fedorov found this type underneath the cyclone track where intense upwelling occurs. The Type A profile is characterized by strong cooling (1°-3°C) in the upper layer of 30-90 m depth and weak warming (0.5°-2.0°C) at 100-200 m. Fedorov attributed Type A profiles to intense vertical mixing and downwelling on the right and left sides of the cyclone track.

The time evolution of the temperature at four levels in the upper ocean was obtained in 1975 when Hurricane Eloise crossed over two buoys in the Gulf of Mexico (Johnson and Withee 1978). The temperature at 2 m depth decreased 2°C immediately after the storm and remained almost unchanged during the following two weeks. At 53 m depth, temperature oscillations with a period close to inertial were detected. The temperature at this level slowly increased over four days until it was equal to the temperature at 2 m depth, which indicated that continued entrainment of water from the thermocline had deepened the mixed layer to at least 53 m.

Detailed information on the three-dimensional structure of the upper ocean in the wake of a tropical cyclone was gathered during the special USSR project Typhoon'75 that involved five research vessels (Pudov et al. 1978). Nine hydrographic surveys to depths of 500-1000 m were made across the track of Typhoon Tess, which had moved at 5-6 m s\(^{-1}\) with an intensity of 48-50 m s\(^{-1}\) in this region. This unique data set revealed that the swath of cooled surface water along the typhoon track was about 400 km wide (Fig. 5.1a). The SST decreased to 23-25°C with a background temperature of 27-28.5°C. The maximum cooling was observed to the right of the typhoon track where the mixed layer deepened to 60-65 m (Fig. 5.1b). The vertical cross-section of temperature change normal to the storm track (Fig. 5.1c) substantiates Fedorov's conclusions on the different types of temperature structures induced by a tropical cyclone. Near the track, the lower temperatures throughout the column indicate upwelling. In the vicinity and beyond of the radius of maximum winds, there was a layer of warmer water at the top of the former thermocline due to downward heat flux as the mixed layer deepened. This layer was relatively thin near the center, but became greater in vertical extent with increasing radius, which may have been due to downwelling on the periphery. Changes in the temperature field were detected to depths of 500 m, which was the first observational evidence of such a large vertical scale of upwelling induced by a moving tropical cyclone.

The next USSR expedition Typhoon'78 (Pudov 1980) investigated the area in which Typhoon Virginia remained quasi-stationary for more than three days. The typhoon executed a looping track in the region centered at 22°N, 147°E. One month before typhoon passage, a detailed oceanographic survey had been made in the area bounded by 22°-28°N, 143°-145°E, which provided the pre-storm ocean structure. As a result of the prolonged forcing, strong upwelling generated a cold water core with temperature decreases of more than 6°C at depths of 20-40 m. Penetration of the density anomalies was observed to depths of almost 1000 m in that case. It was also found that a well-defined cyclonic, cold-core eddy was created with a structure and energy budget similar to Gulf Stream rings (Ivanov et al. 1980). The eddy formation process resulting from the mass and velocity field adjustments was later simulated numerically by Ginis and Dikinov (1989).

An excellent review of 10 years of ocean temperature observations from 1971-1980 has been made by Black (1983). Whereas previous temperature observations had been obtained either from buoys and hydrographic surveys, Black obtained the SST data from airborne infrared radiation thermometers (AIRTs) and aircraft-expendable bathythermographs (AXBTs) deployed from the NOAA WP-3D research aircraft. His summary of many cases has a crescent-shaped pattern for the SST decrease underneath a tropical cyclone with the largest decrease located in the right-rear quadrant between R\(_m\)
Fig. 5.1 (a) SST around the track of Typhoon Tess, which moved toward the northwest at 6 m s\(^{-1}\). Observations were made at 20 km intervals along five sections 3-5 days after the storm passage. Note that the minimum in SST occurred 50-150 km to the right of the track. (b) Temperature (°C) along section AB in (a). The base of the mixed layer is shown as a heavy dashed line. Note the 200 km wide, 40 m amplitude upwelling beneath the track. (c) Temperature changes (contour interval, 0.5°C; positive, hatching) along the section AB relative to the climatological values (Pudov et al. 1978).

and 2R\(_m\), where R\(_m\) is the radius of maximum winds. Black concluded that the maximum SST decreases were primarily a function of the storm translation speed. Storms moving faster (slower) than 3 m s\(^{-1}\) produce 1-3°C (3-5°C) within one-half day after storm passage. An additional SST decrease of 1.2-2.0°C was observed in the time range of 0.6-1.4 days after storm passage. SST decreases appeared to be a function of storm intensity only for storms moving slower than 3.5 m s\(^{-1}\) and maximum winds less than 40 m s\(^{-1}\). For more intense storms, the SST decreases were almost independent of storm intensity.
One case analyzed by Black (Hurricane Ella in 1978, Fig. 5.2) also had hydrographic observations by the Russian research vessel Akademik Kurchatov at nearly the same time (Fedorov et al. 1979). Two days before the hurricane, a detailed hydrographic section along 69.5°W was made and these data served as background information on the ocean structure in this region. As indicated by the track in Fig. 5.2, Ella was moving rapidly to the northwest when it stalled and then quickly turned sharply to the northeast. The SST decrease in the wake of Ella based on Black’s AXBTs reached 4-5°C in the area where the hurricane moved slowly. The cross-track temperature deviations obtained by the ship the day after the hurricane passage are shown in Fig. 5.3a. The overall pattern of temperature changes was very similar to that in the Typhoon Tess (compare Fig. 5.1c). Because Ella was a relatively weak and rapidly moving storm when it passed over the ship survey region, the maximum SST decrease was about 1.7°C and occurred 30 km to the right of the track. The rightward bias in the mixed-layer depth increases is also well identified. The narrow column of cooler water at depths of 30 - 50 m in the center of the cross-section indicates the effect of intense upwelling. Twenty days after the passage of Ella, a third hydrographic survey (Fig. 5.3b) was made at exactly the same stations. The striking result from these measurements is that the main qualitative features of the temperature deviations remained
almost the same, although the whole structure was shifted about 20 miles to the south. This data set provided clear and direct evidence for the persistence of the temperature anomalies produced by tropical cyclones. Using this dataset and results of numerical modelling, Ginis and Dikinov (1981a) suggested some mechanisms of slow relaxation of a hurricane's wake in the ocean based on geostrophic adjustment theory.

Using satellite infrared imagery, Stramma et al. (1986) analyzed 13 hurricane wakes in the western North Atlantic ocean. The maximum SST decrease for most of the observed cases was found to the right of the storm track, which is consistent with the hydrographic surveys. The maximum SST decrease of 3.5°C is somewhat smaller than the maximum decrease for the cases discussed above. Stramma et al. speculated the smaller SST decrease may be due to the deep mixed layer and seasonal thermocline in this region.

The ocean response under two 1984 hurricanes (Norbert and Josephine) was studied by Sanford et al. (1987) Airborne eXpendable Current Profilers (AXCPs) deployed from the NOAA WP-3D aircraft. For the first time the horizontal and vertical structures of the ocean temperature and currents were observed in the direct storm-forced region. The largest increase in mixed-layer depth and SST decrease (about 2.2°C) occurred on the right side of the Hurricane Norbert, which moved with the average speed of 3.7 m s⁻¹. This rightward bias is in agreement with previous observations. The measurements in Josephine were in a region with relatively large pre-existing horizontal thermal gradients, and therefore the SST field did not indicate a clear hurricane-related response. The Norbert and Josephine surveys of velocity profiles will be discussed in the next section.

Black et al. (1988a) reported a successful aircraft deployment of three drifting buoys ahead of Hurricane Josephine (1984). The deployment resulted in detailed simultaneous measurements of surface wind speed, surface pressure and subsurface ocean temperature during and subsequent to storm passage. Buoy trajectories and temperature measurements at 40, 60, and 100 m levels revealed the existence of a series of cyclonic and anticyclonic mesoscale eddies in the region of the storm. The ocean response to hurricane passage was superposed on the preexisting eddy field, which made the interpretation of the observations
rather difficult. Nevertheless, the temperature measurements indicated some plausible signatures of hurricane forcing. A maximum mixed-layer temperature decrease of 1.8°C was observed to the right of the storm track. A temperature increase of 3.5°C at 100 m and subsequent decrease of 4.8°C following storm passage indicated a combination of turbulent mixing, upwelling, and horizontal advection processes.

More recently, Shay et al. (1992) reported comprehensive observations from the NOAA WP-3D research aircraft during the passage of Hurricane Gilbert (1988) in the Gulf of Mexico. This experiment improved upon previous efforts in measuring the current and temperature structure before the hurricane and one and three days following the storm passage. A total of 78 AXCPs and 60 AXBTs were successfully deployed. In addition to flight-level meteorological observations, the near-surface wind stress distribution was obtained from the Stepped Frequency Microwave Radiometer (SFMR). Pre-storm thermal structure was characterized by nearly uniform and warm surface temperature of 29°C. Gilbert moved over the survey area with an average speed of 5.6 m s\(^{-1}\) and had maximum sustained winds of 50-55 m s\(^{-1}\) at flight level. As a result of storm forcing, the SST decreased markedly in the area surrounding the eye (Fig. 5.4). A maximum decrease of 3.5-4.0°C was observed 1.5 days following the storm passage. The region of enhanced cooling extended about 2R\(_m\) - 3R\(_m\) (150 km) to the right of the storm, which is consistent with previous data. The storm passage increased the mixed layer depth from pre-storm values of 30-35 m to approximately 60 m on the right side of the track. This mixed layer deepening was associated with enhanced vertical turbulent mixing in the upper ocean layer. Continued evolution of the cross-track thermal structure was found between 36 hours (Wake I experiment) and 80 hours (Wake II experiment) after the storm passage (Fig. 5.5) in association with three-dimensional dynamical processes induced by Hurricane Gilbert in the upper ocean. A relative minimum of 25-30 m in the mixed layer depths along the track was attributed to the upwelling induced by the wind-driven current divergence behind the eye. The increase of the mixed layer depth to 60-80 m at 1-3R\(_{max}\) during Wake II was probably a result of the combined effect of vertical mixing and downwelling caused by convergence of the mixed layer currents. Numerical simulations of these changes in the thermal structure of the upper ocean will be discussed in Chapter 5.4.

5.2.2 Ocean currents induced by tropical cyclones

The data base of ocean current observations generated by tropical cyclones is much smaller than the temperature data base. Most current observations are from buoys and mooring arrays that happened to be located near the hurricane passage. Probably the first velocity structure measurements in a hurricane wake were in the Gulf of Mexico during the passage of the Hurricane Eloise (Withee and Johnson 1976). Generation and rapid decay of near-inertial motion was observed at 53 m depth (roughly the mixed layer) during and after the passage of the hurricane. Wind generation of near-inertial motions has been well established since the pioneering studies of Veronis (1956) and Veronis and Stommel (1956) and has been demonstrated for tropical cyclones in several analytical and numerical models, as will be described in the next sections.

Mayer et al. (1981) presented measurements of the current response to Hurricane Belle, which passed over the New York Bight in 1976. The observations indicated intense near-inertial oscillations even in shallow water depths of 75 m. A 180° phase reversal of the oscillations was observed across the mixed-layer depth, which suggested the dominance of a first baroclinic mode response. Brooks (1983) described the internal near-inertial signal in a deeper water from three moorings deployed at depths between 200 and 700 m during the passage of Hurricane Allen. The near-inertial current amplitudes as 700 m were 15 cm s\(^{-1}\). These were the first observational evidences of the hurricane-induced near-bottom current response in the near-inertial wave band.

A more comprehensive set of ocean current observations was collected during the Hurricane Frederic passage in 1979 (Shay and Elsberry 1987). The hurricane passed within 80 to 130 km of the U.S. Naval Oceanographic Office current meter arrays CMA1, CMA2

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Fig. 5.4 Objective analysis of SST (°C) (left panels), distribution of data points and normalized error (center panels), and V SST (°C) (right panels) derived from the AXCPs and AXBTs in the (a) Pre-storm, (b) Storm, (c) Wake I, and (d) Wake II observation periods in Hurricane Gilbert in a storm coordinate system scaled by $g$ and $R_m$. The location and direction of movement of Gilbert are depicted in the storm frames. Shaded areas in the storm, Wake I, and Wake II depict regions of maximum cooling of 3.5-4°C (Shay et al. 1992).
Fig. 5.5 Cross-track temperature profiles (degrees Celsius) from Wake I (solid curve) and Wake II (dashed curve) observation periods with the cross-track distance relative to the storm track scaled by $R_m$ (50 km). Each temperature profile is displaced by 4°C going from $-R_m$ to about $5R_m$, with a temperature scale in the lower left corner (Shay et al. 1992).

and CMA3 in water depths ranging from 100 to 470 m, and within 150 km of an Ocean Thermal Energy Conversion (OTEC) mooring in 1050 m water depth. Near-inertial wave excitation was observed throughout the water column at the three arrays as the hurricane approached (Fig. 5.6). The mixed layer currents oscillated with 80-90 cm s$^{-1}$ amplitude for about 4 inertial periods (IP) (inertial period is equal to $2\pi/f$, where $f$ is the local Coriolis parameter) and then rapidly decreased. In contrast, the magnitude of the thermocline currents gradually increased over a period of 7 IP, which indicated an energy transfer to the interior. The thermocline temperature at CMA2 and CMA3 also exhibited inertial oscillations. It is important to note that the thermocline currents at CMA3 persisted for nearly the entire record (21 IP) following the storm passage. Thus, the energy from a hurricane that penetrates below the mixed layer will decay very slowly because the turbulent diffusivity is very small. The near-bottom current measurements at CMA3 site indicate near-inertial oscillations (Fig. 5.6a) with amplitudes of about 20 cm s$^{-1}$. Shay and Elsberry (1987) demodulated the current meter observations and expanded the data into a depth-averaged current and the first two baroclinic modes. They found a near-inertial signal in the depth-averaged currents. In the following theoretical and numerical study (Shay et al. 1990), this signal was attributed to the time-dependent barotropic response induced by the near-inertial oscillations of the sea surface elevation. However, this interpretation is controversial. Ginis and Sutyrin (1995) argue that in shallow water, tropical cyclones may generate an energetic near-inertial current response associated with the first baroclinic mode in the near-bottom layer, so that the near-inertial oscillations at the sea surface may be the result of baroclinic effects and may not be associated with the barotropic currents.
5.6 Along-track velocity (cm s\(^{-1}\), solid) and temperature (°C, dashed) time series from moored arrays (depth in meters) at (a) CMA3, (b) CMA2, and (c) OTEC sites from 7 to 25 September 1979. Time \(t = 0\) corresponds to the time of closest approach of Hurricane Frederic at 21 UTC 12 September for CMA2 and CMA3 and at 12 UTC 12 September for the OTEC site (Shay and Elsberry 1987).

All of the current observations discussed above have been made on the continental margin and therefore might be influenced by the shelf-slope topography. Brink (1989) presented the data from a current meter mooring in the western North Atlantic located about 100 km from the track of Hurricane Gloria in 1985. Since the main thermocline extends down to about 1000 m in that area and the total depth is about 5500 m, the mooring was well removed from any topographic influences. After the hurricane passed with a translational speed of 6.5 m s\(^{-1}\), vigorous near-inertial current variability at 159 m with an amplitude of about 15-20 cm s\(^{-1}\) was measured for approximately 20 days. Although the near-inertial response at greater depths was weaker, the signal was detectable even as deep as 1059 m. Thus, the velocity observations obtained from the moored current meters gave valuable information on the generation and evolution of the current response to tropical cyclone forcing. Thus, a tropical cyclone is an efficient mesoscale atmospheric forcing to produce an effective energy flux to the deep ocean layers. However, current meter moorings generally lack sufficient vertical and horizontal resolution as the measurements are only at fixed levels and at a single site.

As indicated above, the first three-dimensional description of the ocean current and temperature response to a hurricane was obtained in Hurricanes Norbert and Josephine in 1984 (Sanford et al. 1987). A NOAA WP-3D aircraft dropped about 15 successful AXCPs for each case with more profiles on the right side of the tracks (Fig. 5.7). Whereas the fast probes descended at a rate of 4.5 m s\(^{-1}\) and sampled currents and temperatures in the upper 1000-1500 m of the ocean, the slower probes descended at 2.2 m s\(^{-1}\) and provided measurements of upper-ocean response to about 200-250 m. The upper-ocean current measurements from an AXCP are normally contaminated by the hurricane-generated surface waves. These low-frequency (0.10 Hz) waves induce orbital velocities ranged between 1-2 m s\(^{-1}\) that decay exponentially with depth. Sanford et al. (1987) developed a three-layer, least-squares model to extract the surface wave components from the original AXCP data. However, this procedure should be applied with caution. In some cases, it has been found that the mean wave-induced currents were in the same direction as the steady
Fig. 5.7 (Upper panels) Track of Hurricane Norbert and the survey region (rectangle) of AXCP launch positions in a storm-centered coordinate system (Price et al. 1994). The hurricane-induced $u$-component (middle panel) and $v$-component (bottom panel) profiles (cm s$^{-1}$) at selected locations as indicated in upper right panel (Shay et al. 1989).
mixed layer currents, which may modulate the vertical current shear and hence vertical mixing processes (Black et al. 1988b). Vertical current profiles from Hurricane Norbert (Shay et al. 1989) provide clear evidence for the storm-induced near-inertial oscillations in the upper ocean. In the right-rear quadrant of Norbert (AXCP 14 in Fig. 5.7), the current velocities were directed northeast, whereas the current velocities were directed southeast in the left-rear quadrant (AXCP 20). This important dynamical effect of hurricane forcing will be discussed in greater detail in the next section.

Shay et al. (1992) described the three-dimensional, upper-ocean current response based on the 78 AXCPs deployed prior to, during, and after the passage of Hurricane Gilbert in the Gulf of Mexico. Cross-track sections of the velocity profiles 36 hours (Wake I experiment) and 80 hours (Wake II experiment) after the storm passage are shown in Fig. 5.8. The most prominent feature of the current response is the significant rightward asymmetry in the current amplitude in the upper-ocean layer. The magnitudes of the current velocities in the mixed layer reached more than 1 m s\(^{-1}\) to the right of the track. Notice also the considerable velocity shear between the mixed layer and the thermocline. The velocity structure observations again demonstrated that the convergence and divergence cycle had a near-inertial period.

![Fig. 5.8 Vector stick plot of the observed velocity profiles (centimeters per second) from (a) Wake I and (b) Wake II observation periods at the same positions as in Fig. 5.5. Each current profile is displaced by 85 cm s\(^{-1}\), going from -R\(_m\) to about 5R\(_m\), with a velocity scale in the lower left corner (Shay et al. 1992).](image-url)
In summary, historical observations are a valuable source for better understanding the physical processes taking place in the ocean during and after hurricane passage. Emerging technological developments in aircraft-based instrumentation have recently provided high-quality measurements of atmospheric and oceanic parameters in the vicinity of a tropical storm in real time. These observations describe many aspects of the ocean response to tropical cyclone, and provide a data base for validating the analytical and numerical models to be described in the next sections.

5.3 THEORETICAL STUDIES OF INTERNAL WAVES GENERATED BY TROPICAL CYCLOONES

As indicated in the previous section, tropical cyclones traveling across the ocean generate a strong baroclinic response that includes three-dimensional, near-inertial internal waves. The structure and evolution of these gravity waves have been studied by many investigators using analytical and numerical models. In this section, the general properties of the wave response to a moving tropical cyclone will be discussed within the framework of linear theory. Some relevant air-sea parameters from five tropical cyclones are included in Table 5.1.

5.3.1 Governing equations

Consider a frictionless continuously stratified ocean of uniform depth \( H \) with a stable density structure \( \rho_0(z) \) and associated buoyancy frequency \( N(z) \). The equations for a hydrostatic and Boussinesq fluid on a \( f \) plane linearized about the background state are

\[
\frac{\partial u}{\partial t} - f v = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + XS(z) \tag{5.1}
\]

\[
\frac{\partial v}{\partial t} + f u = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + YS(z) \tag{5.2}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{5.3}
\]

\[
\frac{\partial p}{\partial t} - \frac{\rho_0 N^2 w}{g} = 0 \tag{5.4}
\]

\[
\frac{\partial p}{\partial z} + \rho g = 0 \tag{5.5}
\]

where \( u, v, \) and \( w \) are velocity components corresponding to the Cartesian coordinates \( x, y, \) and \( z \) measuring distance eastward, northward, and upward respectively; \( f \) is the Coriolis parameter, \( p \) and \( \rho \) represent perturbations of pressure and density from the state of rest, and \( g \) is the acceleration of gravity. \( X \) and \( Y \) are the wind forcing functions \((X, Y) = (\tau_x / \rho_0 H_{mix}, \tau_y / \rho_0 g H_{mix})\) distributed as a body force within the surface mixed layer of thickness \( H_{mix} \); \( S(z) \) is a step function that is unity in the mixed layer and zero elsewhere. The boundary conditions at the sea surface and the ocean bottom are

\[
w = \frac{\partial \eta}{\partial t}, \quad p = \rho_0 \eta \quad \text{at} \quad z = 0 \tag{5.6}
\]

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<th>Parameter</th>
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<td>47</td>
</tr>
<tr>
<td>$L/R_{1}$ (L=2$R_{\text{max}}$)</td>
<td>1.4</td>
<td>1.6</td>
<td>2.4</td>
<td>2.2</td>
<td>1.9</td>
</tr>
<tr>
<td>$\Lambda$ (km)</td>
<td>580</td>
<td>464</td>
<td>363</td>
<td>588</td>
<td>600</td>
</tr>
</tbody>
</table>

\[ w = 0 \quad \text{at} \quad z = -H \quad (5.7) \]

where $h$ is the sea surface vertical displacement.

5.3.2 Normal mode expansion

Solutions of (5.1)-(5.5) can be sought by expanding the variables in terms of noninteracting vertical normal modes (Gill 1982). This method has been widely used in analytical studies of the ocean response to traveling storms (Gill 1984; Kundu and Thomson 1985; Shay et al. 1989; Nilsson 1995). The normal mode expansions are given by

\[ (u, v, p) = \sum_{n=0}^{\infty} (u_{n}, v_{n}, \rho_{0} g h_{n}) \hat{p}_{n}(z) \quad (5.8) \]
\[ w = \sum_{n=0}^{\infty} \frac{\partial h_n}{\partial t} \hat{h}_n(z) \]  \hspace{1cm} (5.9)

\[ S = \sum_{n=0}^{\infty} \sigma_n \hat{p}_n(z) \]  \hspace{1cm} (5.10)

where the expansion coefficients \( u_n, v_n, \) and \( h_n \) are functions only of \( x, y, \) and \( t, \) and \( p_n \) are the coefficients in the expansion of the step function \( S. \) The normal mode eigenfunctions satisfy

\[ \hat{p}_n = \frac{c_n^2}{g} \frac{d\hat{h}_n}{dz} \]  \hspace{1cm} (5.11)

\[ \frac{d\hat{p}_n}{dz} = \frac{N^2}{g} \hat{h}_n \]  \hspace{1cm} (5.12)

with the two boundary conditions

\[ \hat{p}_n = \hat{h}_n \quad \text{at} \quad z = 0 \]  \hspace{1cm} (5.13)

\[ \hat{h}_n = 0 \quad \text{at} \quad z = -H \]  \hspace{1cm} (5.14)

where \( c_n \) is the eigenvalue for the \( n \)th mode and has the dimensions of velocity. The equations (5.11) and (5.12) can then be reduced to a single equation for \( h_n \) as

\[ \frac{d^2\hat{h}_n}{dz^2} + \frac{N^2}{c_n^2} \hat{h}_n = 0. \]  \hspace{1cm} (5.15)

Equation (5.15) is of the Sturm-Liouville form, for which the various solutions are orthogonal. For a continuously stratified ocean, an infinite sequence of eigenvalues \( c_n \) and corresponding eigenfunctions \( h_n \) and \( p_n \) exists. The constant \( c_n \) is regarded as the speed of long gravity waves. Values of \( c_n \) usually range from 2 to 3 \( \text{m s}^{-1} \) for \( c_1, \) and decrease for larger \( n. \)

Substitution of (5.8)-(5.10) into (5.1)-(5.5) gives the equations governing the expansion coefficients

\[ \frac{\partial u_n}{\partial t} - f v_n = -g \frac{\partial h_n}{\partial x} + X_n \]  \hspace{1cm} (5.16)

\[ \frac{\partial v_n}{\partial t} + f u_n = -g \frac{\partial h_n}{\partial y} + Y_n \]  \hspace{1cm} (5.17)

\[ \frac{\partial h_n}{\partial t} + H_n \left( \frac{\partial h_n}{\partial x} + \frac{\partial h_n}{\partial y} \right) = 0 \]  \hspace{1cm} (5.18)
where $H_n$ is the equivalent depth that is related to the eigenvalue $c_n$ by $c_n^2 = gH_n$, and $(X_n, Y_n) = (\sigma_n X, \sigma_n Y)$ is the forcing function projected onto each mode. The equations (5.16)-5.18 are identical to the equations governing the motion of a shallow homogeneous fluid. Therefore, each normal mode in a stratified flow behaves similar to a homogeneous layer of depth $H_n$. As shown in Shay et al. (1989), $X_n$ and $Y_n$ decrease as the mode number increases. This implies that the lower order baroclinic modes are more energetic than the higher order modes.

The $n = 0$ mode is the barotropic mode for which the solution is obtained from (5.11)-(5.15) when $N = 0$

$$\hat{p}_0 = 1, \quad \hat{h}_0 = 1 + z/H, \quad c_0^2 = gH. \quad (5.19)$$

For this mode, the vertical structures of $u$, $v$, and $p$ are depth-independent. The corresponding structure for $w$ is linear in $z$, with zero at the bottom and a maximum amplitude at the sea surface.

The remaining modes with $n \geq 1$ are baroclinic and are found by solving (5.15) using observed density profiles. An example of eigenfunctions computed by Shay (1994) based on a density profile in the Gulf of Mexico is shown in Fig. 5.9. For baroclinic modes, the surface condition (5.13) is usually replaced by the rigid lid condition

$$\hat{h}_n = 0 \quad \text{at} \quad z = 0 \quad (5.20)$$

This approximation can be made because the sea surface vertical displacements are much smaller than those in the ocean interior, and therefore the baroclinic modes are negligibly distorted by the condition (5.20). However, the rigid lid approximation does not imply that

![Fig. 5.9 Normalized amplitude of the vertical (left panel) and horizontal velocity eigenfunctions (right panel) for the barotropic mode (solid) and baroclinic modes 1 (dashed), 2 (dotted), 3 (chain-dotted), and 4 (chain-dashed) based on a $N^2(z)$ profile in the Gulf of Mexico (Shay 1994).]
the free surface displacements corresponding to the baroclinic modes are zero. The magnitude of the surface displacement can still be found from the rigid lid solution because \( \tilde{p}_n \) is not zero at \( z = 0 \) (Fig. 5.9) and (5.6) gives an estimate of \( \eta \). Excluding the wind waves, much of the sea surface displacement in the deep ocean generated by a tropical cyclone is due to baroclinic motion. The barotropic mode has a rather different character. If the rigid lid approximation is applied, the barotropic mode has \( c_0 = \infty \), which yields instant adjustment with a barotropic pressure gradient (Ginis and Sutyrin 1995).

### 5.3.3 Properties of internal waves in the wakes of tropical cyclones

Some important properties of the internal waves excited by a traveling tropical cyclone can be inferred as simple consequences of the dispersion relation of inertia-gravity waves. To derive the dispersion relation, the wave solutions are sought as a complete set of Fourier-mode solutions of the unforced version of (5.16) - (5.18) of the form \( \exp [i(\omega t - lx - my)] \), where \( l \) and \( m \) are the wave numbers in the \( x \)- and \( y \)-directions, respectively, and \( \omega \) is a frequency. Since the system of equations becomes algebraic, the following dispersion relation may be obtained

\[
\omega [\omega^2 - f^2 - c_n^2 k^2] = 0
\]  
(5.21)

where \( k^2 = l^2 + m^2 \). The first root, \( \omega = 0 \), corresponds to a steady, geostrophically balanced flow. The remaining two roots of the equation

\[
\omega^2 - f^2 - c_n^2 k^2 = 0
\]  
(5.22)

correspond to inertia-gravity waves. The phase velocity of the wave is \( c = \omega/k \) and the group velocity with which the wave energy propagates \( C_g = \partial \omega / \partial k \). From (5.22), we get

\[
C_p = c_n (1 + (f/c_n k)^2)^{1/2}, \quad C_g = c_n (1 + f^2/(c_n k)^2)^{-1/2}
\]  
(5.23)

Consider a tropical cyclone moving in the negative \( x \)-direction with a constant velocity \( U_h \) and a wave pattern that is steady in a frame of reference in which the storm is at rest. Any wave crests in this steady field must be stationary. It is evident that since \( C_p \) always exceeds \( c_n \), no stationary waves exist with respect to the forcing if \( c_n > U_h \). By contrast, \( C_g \) is always less than \( c_n \) and therefore the wave field will be confined in a wake behind the storm. The internal wake has a wedge-shaped pattern, analogous in some respects to the surface gravity wave wake behind a steadily moving ship (Lighthill 1987). Tropical cyclones generate predominately waves with wavelengths larger than the Rossby radius of deformation \( R_n = c_n/f \) (Shay et al. 1989). According to (5.23), the group velocities of these waves are small and therefore the wave energy will disperse laterally very slowly after the storm passage.

Inertia-gravity waves that are steady relative to the tropical cyclones must have a phase velocity component in the along-direction equal to \( U_h \), and therefore have frequencies

\[
\omega = U_h l
\]  
(5.24)

The values of \( \omega, l, \) and \( m \) for these waves can be found by combining (5.22) and (5.24). Real solutions are only possible if \( U_h > c_n \). Intersection lines of the plane (5.24) with the surface (5.22) may be projected (Fig. 5.10) onto the wave number plane \((l, m)\)

\[(U_h l)^2 - f^2 - c_n^2 (l^2 + m^2) = 0.
\]  
(5.25)
The wave number curves calculated by Nilsson (1995) in Fig. 5.10 specify the waves for which the phase velocity is such that the wave crests are stationary relative to the forcing field. The longest waves (the waves with lowest wavenumber) are excited in the along-track direction with wavelength \(2 \pi f \left(U_h^2 - c_n^2\right)^{1/2}\). Away from the storm track, the waves have higher wave numbers and therefore propagate with smaller phase speeds, but larger group velocities. The waves in the wake propagate in the directions normal to the wavenumber curves. They fill a wedge between the normals to the asymptotes of the hyperbola (5.25). The semi-angle of the wedge is then given by

\[
\Theta = \tan^{-1}\left(U_h^2/c_n^2 - 1\right)^{-1/2}.
\] (5.26)

The wider (narrower) wedges are associated with smaller (larger) values of \(U_h/c_n\). It is noteworthy that (5.26) can be derived more simply from a right-angled triangle with the hypotenuse \(U_h\) and the side \(c_n\). This follows from the consideration that the wave energy generated by the storm at any given point travels radially a distance \(C_g t\) during a time period \(t\) while the storm has traveled a distance \(U_h t\). The boundary of the wedge is formed by the waves having the largest group velocity, which is equal to \(c_n\) according to (5.23).

![Wave number curves from equation (5.25) representing near-inertial waves in the wake of a tropical cyclone moving to the left with a velocity \(U_h\); I \(U_h/c_n = 1.2\), II \(U_h/c_n = 1.4\), and III \(U_h/c_n = 2.2\). The arrows indicate the direction, relative to the storm, in which waves of each wave number are found (Nilsson 1995).](image)

5.3.4 Horizontal and vertical structures of the tropical cyclone wave wake in the ocean

The set of equations (5.16)-(5.18) can be reduced to a single equation for the vertical velocity \(w_n = \partial h_n/\partial t\) of the \(n^{th}\) baroclinic mode (Shay et al. 1989)
\[ c_n \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c_n^2} \frac{\partial^2}{\partial t^2} - \alpha_n^2 \right] \omega_n = f \nabla \times \tau_n + \frac{\partial}{\partial t} \nabla \cdot \tau_n \]  

(5.27)

where \( \alpha_n = f/c_n \) is the inverse Rossby deformation radius, \( \nabla \times \tau_n = \partial Y_n/\partial x - \partial X_n/\partial y \), and \( \nabla \cdot \tau_n = \partial X_n/\partial x + \partial Y_n/\partial y \) are the wind stress curl and the wind stress divergence, respectively.

If the storm is moving steadily, the local time derivative can be transformed into a space derivative and (5.27) is solved using the Fourier transforms. Geisler (1970) found analytical solutions of (5.27) for a two-layer linear model with impulsive forcing. He emphasized that the nature of the baroclinic and barotropic responses is quite different because of the difference in the long gravity wave speed. Mathematically, the differential operator governing the dynamical response is elliptical when \( U_h/c_n < 1 \) and hyperbolic when \( U_h/c_n > 1 \). The translation speed of a tropical cyclone is typically greater than the baroclinic wave speed \( (U_h > c_1) \) and smaller than the barotropic wave speed \( (U_h < c_0) \). As a result, the characteristic feature of the baroclinic response is an oscillating narrow wake behind the storm formed by slow propagating near-inertial gravity waves. By contrast, the barotropic response consists of a geostrophic current elongated in the along-storm track direction and an associated trough in the sea-surface height. Because Geisler (1970) analyzed the barotropic response using a localized source of positive wind stress curl, very broad currents and sea surface depression were produced on the spatial scale of the barotropic radius of deformation. He then concluded that the barotropic flow must be affected by variation of the Coriolis parameter, and therefore should break down into planetary waves in about one day. However, the recent studies of Shay et al. (1990) and Ginis and Sutyrin (1995) indicate that for more realistic wind stress profiles typical of tropical cyclones, the cross-track horizontal scale of the barotropic current and sea-surface elevation is comparable to the storm size, not to the barotropic radius of deformation. In the deep open ocean, the barotropic currents generated by a hurricane are of the order of 0.02 m s\(^{-1}\), and thus are considerably weaker than the baroclinic currents generated in the surface mixed layer and thermocline, which are on the order of 1 m s\(^{-1}\).

The radius of maximum winds in a tropical cyclone is typically of the same order as the baroclinic radius of deformation. Therefore, impulsive forcing cannot be applied for analyses of the baroclinic response. For a more realistic description of the internal wave wake structure, Geisler (1970) used a wind stress profile that resembled an actual tropical cyclone and solved (5.27) numerically. The vertical velocity field calculated for the storm with the parameters \( R_m/R_n = 4 \) and \( U_h/c_1 = 5 \) is shown in Fig. 5.11. Baroclinic response consists of near-inertial waves and a geostrophic current associated with the semi-permanent baroclinic ridge in the along-track direction. Geisler also shows that the amplitude of the waves (ridge) decreases (increases) for the slower moving storms. The wave wake narrows with an increase of the ratio \( R_m/R_n \) because the wave energy is radiated with a group velocity (5.23) which will decrease for larger wavelengths in the cross-track direction.

Whereas Geisler (1970) analyzed a two-layer model, continuously stratified linear models use the expansion in vertical normal modes (5.8)-(5.10) to represent the ocean vertical structure. Such models have been successfully applied in analyses of the vertical propagation of internal energy from the surface to the ocean interior as a result of moving storms. Gill (1984) studied the internal wave wake in a deep ocean after the passage of a fast-moving storm in the x-direction assuming that the current behind is independent of x. The potential vorticity initially injected into the surface mixed layer by the storm is dispersed horizontally in the y-direction and vertically due to the Rossby adjustment process. In the early stages, rapid loss of energy from the mixed layer goes primarily into the first few baroclinic modes. The rate of loss is proportional to the square of the ratio of the storm length scale to the Rossby radius \( R_n \). Gill (1984) estimated that the time scale...
for the energy to escape from the mixed layer, defined as the time scale over which each mode develops a \( \pi/2 \) phase difference, is about two days. Over a long period of time, energy transfers both to and from the mixed layer, so that mixed-layer energy may increase about as rapidly as it was initially lost.

Kundu and Thomson (1985) studied a wave field in a shallow (\( H=250 \) m), stratified ocean due to a front aligned in the \( y \)-direction and moving in the \( x \)-direction. The assumption of independence perpendicular to the direction of propagation was made and a two-dimensional \((x, z)\) baroclinic solution was considered. In agreement with Gill's results, Kundu and Thomson (1985) found a continuous exchange of energy between the surface mixed layer and the stratified interior, which leads to an intermittency of the near-inertial oscillations. The blue shift, defined as \( \omega/f-1 \), increases with depth, which indicates that the higher frequencies (lower modes) disperse earlier from the mixed layer.

Shay et al. (1989) extended Geisler's study to include a continuously stratified fluid with wind forcing superposed onto the first ten baroclinic modes. The model wind stress was based on the observed parameters of Hurricane Norbert in 1984. The forcing functions \( X_n \) and \( Y_n \) in (5.27) were projected onto the modes using the observed \( N(z) \) profile during the hurricane passage. The simulated horizontal structure in the hurricane wake (Fig. 5.12a) is qualitatively similar to Geisler's solution (Fig. 5.11). The upward (downward) vertical velocities signify areas of upwelling (downwelling) associated with near-inertial waves generated by the storm. The pattern broadens in the wake according to (5.26). Shay et al. analyzed the role of the wind stress curl and divergence in the baroclinic response. The wind stress divergence terms were omitted in the Geisler (1970) study. Although the wind stress curl dominated the ocean current response, the wind stress divergence contributed about 25% of the curl term in the direct forcing region if the inflow angle was 20\(^{\circ}\). Whereas the effect of the wind stress divergence damps rapidly, the curl has a more persistent effect.

The simulated vertical structure of the along-track velocity component of the ocean current in Shay et al. (1989) study is shown in Fig. 5.12b. Within the first 1.5 IP, the maximum velocity decreased from 1.3 m s\(^{-1}\) to about 0.8 m s\(^{-1}\) as the contribution of the
first baroclinic mode became out of phase with the remainder of the solution in the mixed layer. At later times of 4.8-5.0 IP, the mixed layer velocity increased by about 0.1 m s⁻¹, presumably because of an exchange of energy between the mixed layer and thermocline, which is the effect predicted by Gill (1974) and Kundu and Thomson (1985). The vertical velocity structure in Fig. 5.12b appears to be rather simple, with reversals in the thermocline current. Because only small differences were found between the observed velocity profiles and the model profiles based on summation of the first ten and first four modes, they conclude that the vertical velocity structure in the tropical cyclone wake can be represented fairly well by a sum of the first several modes.

5.4 NUMERICAL MODELING OF THE OCEAN RESPONSE TO A TROPICAL CYCLONE

Although theoretical studies provide considerable insight into the dynamical processes that govern the three-dimensional internal wave structure behind a moving storm, analytical solutions are generally possible only for a highly simplified wind stress pattern and a fixed vertical structure. In addition to generating near-inertial currents, tropical cyclones force non-oscillatory currents that may have an amplitude comparable to that of the near-inertial currents. Turbulence generated by the wind stress, convective overturning, and velocity shear across the mixed layer base causes entrainment mixing that deepens the mixed layer. Thus, the ocean response to a hurricane is a combination of strongly nonlinear, three-dimensional processes accompanied by large amplitude variations of thermodynamic quantities. A full evaluation of the problem for conditions representative of real tropical cyclones requires a very extensive numerical calculation. In this section, numerical simulations of the ocean response based on the full system of primitive equations will be presented. Most of these simulations have focused on the thermodynamic processes in the upper ocean where turbulent mixing, upwelling, and inertia-gravity oscillations dominate. The main focus of this section will be on these
thermodynamic processes because they are associated with SST changes that may affect the tropical cyclone.

5.4.1 Characteristics of numerical models

Two major processes must be resolved in modeling the ocean response to hurricane forcing: mixed layer thermodynamics and internal gravity wave dynamics. Both processes generate significant amplitude responses. The first process is mainly a local turbulent momentum and heat exchange with the atmosphere at the surface and with the thermocline below by entrainment at the mixed layer base. The second process is inherently a non-local horizontal and vertical energy dispersion by internal-inertial waves. Two types of models have been used to simulate the ocean response to a hurricane: layer models (Elsberry et al. 1976; Chang and Anthes 1978; Sutyrin 1980; Ginis and Dikinov 1981a,b; Price 1981, 1983; Greatbatch 1983, 1984; Ginis and Dikinov 1989; Cooper and Thompson 1989a), and level models (Chang 1985; Shay et al. 1990; Price et al. 1994). A brief discussion will be given of the main features of the layer and level models.

5.4.1.1 Layer models. The distinguishing feature of the layer models is the treatment of the surface mixed layer as a layer for both dynamics and thermodynamics. The mixed layer is viewed as a turbulent well-mixed layer with vertically uniform temperature, salinity, and currents. Immediately below the mixed layer, the rate of turbulent mixing is small, which results in large gradients of temperature and salinity between the mixed layer and the top of the thermocline. This transition layer plays a key role in the dynamics of the mixed layer because non-advective deepening of the mixed layer occurs by entrainment of thermocline fluid across this stratified layer. Thus, turbulent mixing is necessarily occurring not only within the mixed layer but also in the transition layer. Representing the character and intensity of this turbulence is the key to accurate modeling of mixed layer physics.

The usual practice is to treat this transition layer as a discontinuity surface, which is referred to as the mixed layer base. An important aspect of the energetics of the mixed layer is the possible radiation of the turbulent energy into the thermocline through the base of the mixed layer. This problem has been studied extensively for the last two decades (a recent review is given by Fernando 1991). No consensus exists on the amount of turbulent energy radiated from the mixed layer. Some theoretical and laboratory analyses suggest that the bulk of the energy fed into the mixed layer is trapped by the transition layer and is eventually dissipated by wave breaking. This condition is probably appropriate for the ocean areas that experience the most intense hurricane forcing, and where sharp gradients of the thermodynamic quantities are usually observed between the mixed layer and the thermocline below. For this reason, it is generally assumed in the layer models that turbulent momentum and heat fluxes do not penetrate below the mixed layer base.

Equations for the mixed layer temperature \( T_1 \), which is also the SST, and depth \( h_1 \), are derived by integration of the continuity and thermodynamic equations from the sea surface to the mixed layer base,

\[
\frac{\partial h_1}{\partial t} = -\nabla \cdot (h_1 \mathbf{v}_1) + w_e, \tag{5.28}
\]

\[
\frac{\partial T_1}{\partial t} = \frac{Q_s}{h_1} - \mathbf{v}_1 \cdot \nabla T_1 - w_e \frac{T_1 - T_2}{h_1}, \tag{5.29}
\]

where \( \mathbf{v}_1 \) is the current velocity vector in the mixed layer, \( w_e \) is the entrainment rate, \( T_2 \) is the temperature at the top of the thermocline, and \( Q_s \) is the net heat flux at the surface divided by \( \rho c_p \).

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In (5.28), the time change in the mixed-layer depth is determined by the horizontal divergence $\nabla \cdot (h_1 v_1)$, which is equal to the velocity component normal to $z = h_1$, and the turbulent mass flux $w_e$ across this surface. Notice that entrainment is not a vertical velocity, as only a mixed-layer depth increase is possible by turbulent mixing, and therefore no layer shallowing (detrainment) is allowed by this process. By contrast, current divergence may be positive (upwelling) or negative (downwelling), which will shallow or deepen the mixed-layer depth, respectively. Thus, regions of upwelling decrease $h_1$ against the deepening associated with entrainment. In regions with downwelling and entrainment, the mixed-layer depth always increases.

The thermodynamic equation (5.29) indicates that the mixed-layer temperature ($T_1$) is determined by the surface heat flux, horizontal advection, and the turbulent heat flux at the mixed layer base. Notice that the vertical advection does not directly influence the SST. However, the reduced mixed layer thickness $h_1$ will contribute to an enhanced SST decrease for the same entrainment heat flux $w_e(T_1 - T_2)$ in the third term of (5.29). In addition, upwelling may increase the entrainment rate by reducing the mixed-layer thickness. Thus, the SST decrease above the upwelling region may be larger than in adjacent downwelling areas. Although the SST decrease and mixed-layer depth increase will be correlated in the mixing-dominated regions, this will not be true in the upwelling-dominated regions. Downwelling, which is usually found on the left and right sides of the storm track, tends to thicken the mixed layer and reduce the effect of entrainment mixing and therefore SST decrease. As will be discussed in the next section, the heat flux at the sea surface and horizontal advection terms in (5.29) have much smaller contributions to SST decreases.

An important parameter in the thermodynamic equation (5.29) is the temperature at the top of the thermocline, $T_2$. In some earlier models (e.g., Chang and Anthes 1978; Sutyrin 1980), this temperature was assumed to be unchanged. However, a proper representation of the layer immediately below the mixed layer is essential because it determines the water properties that are entrained into the mixed layer. The equation for the temperature $T_2$ can be written as

$$\frac{\partial T_2}{\partial t} = -v_2 \cdot \nabla T_2 - w_e \frac{T_2 - T_3}{h_2} \gamma,$$

where $v_2$ and $h_2$ are the velocity and thickness of the stratified layer immediately below the mixed layer, $T_3$ is the unchanged temperature at the base of the layer, and $\gamma$ is a parameter that depends on the layer stratification. The stratification is specified either as a linear function ($\gamma = 1$) (Price 1981; Cooper and Thompson 1989a; Bender et al. 1993) or as a parabola ($\gamma = 2$) (Ginis and Dikinov 1981a,b; Ginis and Dikinov 1989; Khain and Ginis 1991). Note that changes of temperature $T_2$ may result only from entrainment and horizontal advection.

Although the density of ocean water is also a function of salinity, the layer models of the ocean response to tropical cyclones have assumed constant salinity. Predictive equations for the salinity variations in the mixed layer and the stratified layers below may be derived in a manner similar to the temperature (e.g., Price 1981).

The vertical structure of a layer model is usually represented by a surface mixed layer and a specified number of layers within the thermocline and deep water. All layer interfaces below the mixed layer are considered to be material surfaces and therefore are isopycnal surfaces. In other words, the temperatures at these interfaces remain unchanged as the interface is raised (lowered) in regions of upwelling (downwelling). The equations for the layer thicknesses derived from continuity equation are written...
\[
\frac{\partial h_2}{\partial t} + \nabla \cdot (h_2 v_2) = -w_e, \tag{5.31}
\]

\[
\frac{\partial h_i}{\partial t} + \nabla \cdot (h_i v_i) = 0, \quad (i = 3, N) \tag{5.32}
\]

where \(N\) is the number of layers.

The equations for the momentum balance in each layer are derived by vertically integrating the momentum equations and taking into account the kinematic and dynamic conditions at the sea surface and at the layer interfaces. They are

\[
\frac{\partial v_1}{\partial t} + v_1 \cdot \nabla v_1 + f k \times v_1 = P_1 + \frac{\tau_s}{h_1} + w_e \frac{v_1 - v_2}{h_1}, \tag{5.33}
\]

\[
\frac{\partial v_i}{\partial t} + v_i \cdot \nabla v_i + f k \times v_i = P_i, \quad (i = 2, N), \tag{5.34}
\]

where \(\tau_s\) is the wind stress vector and \(P_i\) is the pressure gradient integrated over the layer \(i\) (\(\tau_s\) and \(P_1\) have kinematic units). The expressions for \(P_i\) vary for different models depending on the type of vertical stratification. Assuming that the temperature below the mixed layer is linearly depth-dependent within each layer, and assuming a flat bottom, they take the form

\[
P_1 = g h_1 [\rho_1 \nabla \xi_1 + \nabla p_a + \frac{1}{2} h_1 \nabla \rho_1], \tag{5.35}
\]

\[
P_i = g h_i [\rho_i \nabla \xi_1 + \nabla p_a + (\rho_i - \rho_1) \nabla h_1 + \sum_{j=2}^{i-1} (\rho_j^* - \rho_j^*) \nabla h_j
\]

\[
+ \frac{1}{6} (\rho_{i+1} - \rho_i) \nabla h_i + h_i \nabla \rho_1 + \frac{1}{2} h_2 \nabla \rho_2], \quad (i = 2, N) \tag{5.36}
\]

where \(\xi_1\) is the sea-surface displacement, \(g\) is the gravity acceleration, \(p_a\) is atmospheric pressure normalized by \(g\), \(\rho_1\) is the mixed layer density, \(\rho_i (i = 2, N)\) are the densities at the layer interfaces, and \(\rho_{i+1}^* = 1/2 (\rho_{i+1} + \rho_i)\). The surface displacement \(\xi_1\) is given by

\[
\xi_1 = H - \sum_{i=1}^{N} h_i \tag{5.37}
\]

where \(H\) is the initial total ocean depth.

The model dynamics involve both baroclinic and barotropic gravity waves. As was stated earlier, the response to a tropical cyclone in a deep open ocean is primarily baroclinic. Integration of the full model would require a very small time step to resolve the rapidly propagating barotropic wave and therefore appears to be impractical. The barotropic mode may be filtered out by using the reduced gravity approximation, which assumes that the lowest model layer is infinitely deep and at rest and that the pressure gradient vanishes. Taking the limit \(h_N \to \infty\) in (5.36), the gradient of the surface elevation is given by
\[ \nabla \xi_1 = -\frac{1}{\rho_N} [\nabla p_e + (\rho_N^* - \rho_1) \nabla h_1 + \sum_{j=2}^{N-1} (\rho_N^* - \rho_j) \nabla h_j + h_1 \nabla p_1 + \frac{1}{3} h_2 \nabla \rho_2]. \] (5.38)

This system of equations is usually closed with a linear equation of state relating the temperature, salinity, and density.

The largest difference between the layered models of tropical cyclone-ocean response is in the parameterization of the vertical turbulent mixing (entrainment). Three mechanisms for turbulent kinetic energy production in the mixed layer may be invoked: production by near-surface, wind-driven shear; velocity shear at the base of the mixed layer; or by convective overturning due to the surface buoyancy fluxes. Scale velocities characterizing these mechanisms are: \( u = (T_s / \rho)^{1/2} \) (friction velocity); \( \delta v = v_{1-2} \) (velocity jump across the mixed layer base); and \( w = (\alpha g h_1 Q_s)^{1/3} \) (free-convection velocity associated with the buoyancy flux at the sea surface). In addition, the velocity scale related to the temperature jump \( \delta T = T_1 - T_2 \) is \( c_j = (\alpha g h_1 \delta T)^{1/2} \). The entrainment rate may be expressed as a function of three nondimensional parameters

\[ \frac{w_e}{V_q} = f(R_i_T, R_i_v, R_i_w), \] (5.39)

where \( V_q \) is a velocity scale, \( R_i_T = \frac{c_1}{u^2} \); \( R_i_v = \frac{c_1}{(\delta v)^2} \) and \( R_i = \frac{c_2}{w} \).

Various weights have been ascribed to these factors. For example, Elsberry et al. (1976) assumed that the mixed-layer turbulence is generated only by the friction velocity and the negative heat flux at the sea surface

\[ \frac{w_e}{u_*} = [mce^{-h_1/z} + R_i]Ri_T^{-1}, \] (5.40)

where \( z = 100 \) m is a length scale, and \( m = 2 \). Chang and Anthes (1978) also ignored the influence of the velocity shear at the base of the mixed layer and used a modified Kraus and Turner (1967) formulation

\[ \frac{w_e}{u_*} = c_{kp} R_i_T^{-1}, \] (5.41)

where the constant \( c_{kp} \) was set to 2.5. In contrast, Price (1981) ignored the influence of the surface fluxes and assumed only the velocity shear mechanism

\[ \frac{w_e}{\delta v} = 5 \cdot 10^{-4} R_i_v^{-4} \] (5.42)

In Ginis and Dikinov (1989), a formula close to (5.40) was used. Cooper and Thompson (1989a) applied alternately both (5.41) and (5.42).

All of the above mechanisms for turbulent kinetic energy production were taken into account in the numerical simulations of Khain and Ginis (1991) and Bender et al. (1993) based on the Deardorff (1983) parameterization. Using a wide range of experimental data, Deardorff constructed a comprehensive mixing (entrainment) parameterization that can be written as

\[ \frac{w_e}{V_q} = \frac{[0.1 - 0.06 \exp(-4R_i_q)](R_i_q/F_h)^{3/2}(R_i_v^{-3/2} + F_{nR_i_v^{-3/2}}) - 0.14}{1.2 + 0.3R_i_q - 0.3[1 - 0.93 \exp(-0.35R_i_q^{-3/2})]R_i_q R_i_v^{-1} F_h^{-1}}. \] (5.43)
where $F_h = 1.8 (1-2^{1/2} f|v_1|h_1 u_*^{-2})$ and $F_h = (1+1.9 R_i q)^{-0.4}$. $R_i q$ is a non-dimensional parameter based on the velocity scale involving the turbulent kinetic energy. Following Deardorff, (5.43) is combined with an empirically obtained formula

$$
\frac{w_e}{V_q} = \begin{cases} 
0.66-0.52R_i q & \text{if } R_i q \leq 0.52 \\
0.28R_i q^{-1.34} - \exp[-5(R_i q - 0.52)] & \text{if } R_i q > 0.52
\end{cases}
$$

(5.44)

and is solved for $R_i q$. The entrainment rate $w_e$ is finally obtained from the identity

$$
\frac{w_e}{u_*} = (R_i q/R_i q)^{1/2} \frac{w_e}{V_q}
$$

(5.45)

The major differences among various entrainment parameterizations can be seen (Fig. 5.13) in the SST anomalies calculated by using a four-layer version of the model (5.28)-(5.38). An idealized hurricane is moved over an initially undisturbed ocean with a translational speed of 5 m s$^{-1}$. It is evident that the cold wake based on the Deardorff’s parameterization (Fig. 5.13c) nicely consolidates the characteristics of the SST patterns produced using the Elsberry et al. (1976) and Price (1981) formulas. Specifically, the maximum SST decrease is very similar to the Price model (Fig. 5.13b) and located at the right side of the track where the turbulent mixing generated by the velocity shear is dominant. At the same time, the SST pattern on the left side of the track agrees well with the Elsberry et al. model (Fig. 5.13a) where surface momentum and heat fluxes contribute predominantly to SST decreases.

The model equations outlined in this section, with different numbers of layers and some additional simplifications, have been used extensively for numerical simulations of the ocean response to tropical cyclones.

5.4.1.2 Level models. The level models by Chang (1985), Shay et al. (1990) and Price et al. (1994) for simulations of the ocean response to a tropical cyclone are based on the standard set of primitive equations on the $f$ plane. Price et al. (1994) additionally simplified the system of equations by applying the reduced gravity approximation and ignoring horizontal diffusion. Although based on the Richardson number ($R_i$), the vertical mixing in these models is parameterized in quite different ways. In the Price et al. (1994) model, vertical mixing is treated using the hybrid mixed layer formulation of Price et al. (1986), which assumes that the upper-ocean density and velocity will be mixed vertically to satisfy the three stability criteria

$$
\frac{\partial \rho}{\partial z} \geq 0,
$$

(5.46)

$$
R_b = \frac{g \delta \phi h}{\rho_0 (\delta V)^2} \geq 0.65,
$$

(5.47)

$$
R_g = \frac{g \partial \rho/\partial z}{\rho_0 (\delta V/\delta z)^2} \geq \frac{1}{4},
$$

(5.48)

where $V$ is the velocity vector. In the Chang (1985) model, the eddy diffusivity ($K_z$) depends on the Richardson number and a specified mixing length ($l_m$) as

$$
K_z = \begin{cases} 
(1 - R_i g)^{1/2} \frac{8V}{\partial z} l_m^2 & R_i g < 1 \\
0 & R_i g \geq 1
\end{cases}
$$

(5.49)
Fig. 5.13 SST anomalies produced by an idealized moving hurricane in the experiments using a four-layer version of the (5.28)-(5.38) layer model with different entrainment parameterizations: (a) Elsberry et al. (1976), (b) Price (1981), (c) Deardorff (1983). The storm is moving from right to left. The crosses indicate the initial and 1 day center positions.
The important advantage of a level model in comparison with a layer model is a better representation of the vertical dynamic and thermodynamic structures. Shay et al. (1990) use a vertical grid with 17 levels for the total depth of 610 m. The highest resolution of 10 m is in the upper ocean increasing to 80 m below the thermocline. The Price (1994) model has 25 levels with a vertical grid spacing of 10 m within the upper 150 m. The grid spacing increases to 50 m and then to 100 m for greater depths down to the base of the thermocline. A disadvantage of a level model is it is not very efficient. Either the near-surface resolution is inadequate, or if a large number of levels are used, each level in the mixed layer repeats the same values. Whereas the above parameterizations of mixed layer physics require a discontinuity at the base of the mixed layer, the numerical treatment in a level model implicitly assumes continuous variations in the vertical. In the Shay et al. (1990) and Price et al. (1994) models, vertical mixing is generated by only current instability, which may be unrealistic for hurricane conditions in which surface friction and buoyancy fluxes may play a rather significant role.

5.4.2 Simulations of the ocean response in layer models

Early numerical analyses of the ocean response to a tropical cyclone assumed a stationary, axisymmetric forcing. O'Brien and Reid (1967) used a two-layer model to simulate an upwelling generated by a hurricane wind stress. The interaction of upwelling and turbulent mixing was studied by Elsberry et al. (1976), Arsenev et al. (1976), and Ginis and Dikinov (1981a,b, 1982). These relatively simple models have been able to simulate some observed features of the ocean response, such as a shallowing of the mixed layer and the formation of a core of cooler water in the thermocline under the central area of hurricane forcing, and the maxima of SST decrease and mixed layer deepening in the areas of maximum winds. However, they are unrealistic since the tropical cyclone motion is excluded.

Chang and Anthes (1978) utilized the two-layer reduced gravity model (5.28),(5.29), and (5.33) to study the ocean response to a moving tropical cyclone. A highly asymmetric current response, with much stronger velocities on the right of the storm track, was simulated. The SST distribution was also rightward biased, although to a much smaller degree. Because the entrainment (5.41) depended only on the wind stress, the asymmetry in SST was explained by the asymmetry of the wind stress relative to the storm track. Increases (decreases) in the translation speed of the storm reduce (increase) the magnitude of the SST decrease, but the maximum current velocities remain about the same.

A more thorough investigation of the ocean response to a hurricane has been performed by Price (1981), who used a four-layer, reduced-gravity model (5.28)-(5.38) to simulate the ocean response to the passage of Hurricane Eloise. The pressure gradient terms below the mixed layer (5.36)-(5.38) were simplified by assuming the gradient of density to be constant in space and time. Based on observations at buoy EB-10, the initial mixed-layer depth was set to 30 m and the gradient of temperature in the layer below was set to 0.125°C/m. The axisymmetric hurricane forcing, which was unchanged in time, was moved with a speed of 8.5 m s⁻¹. The maximum wind speed was 33 m s⁻¹ at a distance 45 km from the storm center. The simulated cross-track currents and SST deviations shown in Fig. 5.14 are strongly biased to the right of the hurricane track. This is consistent with the simulations of Chang and Anthes (1978).

Price explained the rightward bias in the current response by an inherent asymmetry in the coupling between the wind stress of a moving hurricane and the wind-driven, mixed-layer velocity. The currents in the model solution tend to rotate inertially (clockwise in the Northern Hemisphere) since they are not balanced by a steady pressure gradient. On the right side of the track, the wind stress associated with the translating storm also turns clockwise with time when viewed from the ocean. For a typical hurricane translation speed, the wind stress rotation rate approximately matches the inertial rotation rate of the current. Because the current and wind stress tend to remain roughly aligned throughout
most of the hurricane passage, the current velocity on the right side of the track is accelerated. On the left side of the track, the coupling between the currents and the wind stress is much less efficient because the wind stress rotates anti-clockwise during the hurricane passage. As a result, the mixed-layer currents on the left side of the track do not become as large as on the right side. Some of the left-to-right asymmetry in the mixed-layer currents may be due to stronger wind stresses on the right side of a moving hurricane. However, the factor of 3-4 difference in current amplitudes across the hurricane track is likely due to the asymmetric rotation of the wind stress discussed above.

In Price's model, parameterization of entrainment (5.42) depends only on the velocity shear across the mixed layer base. As a result, the asymmetry in the current field leads to the asymmetry in the SST field. Another type of mixing parameterization might lead to a different SST response (see Fig. 5.13). Price also pointed out that entrainment dominated the mixed layer thermodynamic budget (5.29). The net surface heat flux $Q_s$ was about ten times smaller than the entrainment heat flux $w_e (T_1 - T_2)$. This result has been confirmed by subsequent numerical studies (e.g., Martin 1982; Ginis and Dikinov 1989; and Bender et al. 1993). However, it should be noted that the surface heat fluxes could make a substantial contribution to turbulent mixing through the convective overturning mechanism. As shown in Fig. 5.13, this mechanism might become important under the forward half and the left quadrants of a hurricane where the wind-driven currents are relatively weak.

The chief features of the dynamical response of the upper ocean to a moving tropical cyclone are illustrated in Fig. 5.15, which is constructed based on a numerical run.
of a four-layer version of (5.28)-(5.38) model using an idealized axisymmetric wind stress to represent the cyclone. The inertially rotating wind-driven currents in mixed layer are highly divergent, which causes a vertical velocity at the base of mixed layer and pumps the fluid in the thermocline up (upwelling) and down (downwelling) at the near-inertial period. It is important to note that the first upwelling cycle of the inertial oscillations shown in Fig. 5.15 occurs well behind the cyclone. This distance may be estimated as

$$\lambda = \frac{\pi - \alpha}{f} U_h,$$

(5.50)

---

Fig. 5.15 Schematic of the upper-ocean dynamic response to a hurricane moving from right to left. The storm center position is indicated by the hurricane symbol. The contour lines in the upper (lower) panel are displacements (m) of the mixed layer depth (18.8°C isotherm) from their initial positions. The dashed lines indicate upward displacements.
where $\alpha$ is the inflow angle (the angle between the wind and the azimuthal direction; a positive angle indicates that the wind blows toward the cyclone center). For slower moving tropical storms and larger inflow angles, the upwelling area is located closer to the storm center. The maximum upwelling in Fig. 5.15 is about 270 km behind the storm center, which is agrees well with (5.50) for the specified parameters: $U_h = 5$ m s$^{-1}$, $\alpha = 90^\circ$, and $f = 5.48 (10^{-5})$ s$^{-1}$. Notice the bow-shaped band of compensating downwelling underneath the central and rear parts of the hurricane and outside of the upwelling zone on the both sides of the storm track. The later feature signifies the horizontal dispersion of near-inertial gravity waves behind the storm.

Vertical motions induced by the divergence of the mixed layer currents produce hydrostatic pressure anomalies within the upper ocean. These pressure anomalies generate currents within the main thermocline. Notice the thermocline currents in Fig. 5.15 are negligibly small underneath the hurricane and reach maximum amplitudes near the area of maximum upwelling. This is probably a characteristic feature of the deep open ocean response to hurricane forcing. Since mixed-layer currents oscillate in time along with the inertial pumping, the resulting thermocline currents also have a near-inertial dependence.

The spatial distribution of upwelling/downwelling also has important dynamical consequences on the mixed-layer depths. The maximum mixed layer deepening of 40 m occurs about 100 km behind the storm center and to the right of the storm track where both intense turbulent mixing and downward motion tend to increase the mixed-layer depth. About 300 km behind the storm, the mixed-layer depth shallows due to strengthening of upwelling and weakening of turbulent mixing.

As discussed previously, upwelling may also influence the turbulent mixing in the mixed layer by making it more effective through decreases in the mixed-layer depth. Downwelling causes the opposite effect. In addition, the pressure-generated thermocline currents below the mixed layer may enhance (weaken) the turbulent mixing by increasing (decreasing) the shear instability across the mixed layer base. Price (1981) investigated the influence of upwelling on the mixing process by omitting pressure gradient and advective terms in the model equations, and varying the storm translation speed. He concluded that the non-entrainment mixing effects had a negligibly small role in the maximum SST decrease for a very rapidly moving tropical cyclone ($U_h = 16$ m s$^{-1}$), but strongly increased (by about 35%) the maximum cooling for a slower moving cyclone ($U_h = 4$ m s$^{-2}$).

The thermocline motions generated by a moving tropical cyclone were further investigated by Price (1983) using a six-layer, reduced-gravity model. Equations (5.28)–(5.38) were simplified by excluding the mixing effects and density depth dependence within each layer. The initially quiescent ocean was forced by a steady moving wind stress typical of tropical cyclones. Kinetic energy of the cyclone-generated currents in the mixed layer rapidly decreased after the cyclone passage mainly due to the energy transfer to the thermocline, rather than due to horizontal dispersion by gravity waves. Thermocline motion generated by this process had a very large vertical scale that exceeded the thermocline thickness. Because the storm moved rather quickly ($U_h = 7$ m s$^{-1}$) in Price's simulations, nearly 80% of the total energy input by the storm went into the kinetic energy of the near-inertial frequency waves. However, a storm also produces a geostrophic flow due to the Ekman pumping effect generated by the wind stress components parallel to the storm track. The partition of the energy between the geostrophic and inertial wave component is highly dependent on the hurricane translation speed (Geisler 1970). For slower moving storms, the geostrophic energy is larger. The three-dimensional structure of the quasi-geostrophic velocity and its persistence needs further investigation.

Greatbatch (1983) studied the effects of the pressure gradients and nonlinear dynamics on the generation of the near-inertial ocean response using a two-layer, reduced-gravity model. He concluded that the pressure gradient terms are small compared to the Coriolis terms on the time scale of a few inertial periods for fast-moving storms ($c_n^2/(fL)^2 << 1$ and $c_n^2/(fL)^2 << 1$, where $c_n$ is the phase speed of baroclinic waves and $L$ is the tropical cyclone length scale). Thus, the dominant balance in the equations of motion is
between the inertia terms and the Coriolis terms. For a slow-moving storm, the pressure gradient terms cause the along-track wavelength to decrease relative to the inertial wavelength.

The ocean response to Typhoon Virginia during 1978 was simulated by Ginis and Dikinov (1989) using a three-layer, reduced-gravity model (5.28)-(5.38). As described in Chapter 5.2, this typhoon produced a strong ocean response as it was quasi-stationary for about 3 days. For the numerical simulations, the initial temperature structure was specified based on the measurements made during the USSR expedition Typhoon 78. The wind stress and heat fluxes were calculated with conventional bulk transfer formulas using specified near sea-surface meteorological parameters. The typhoon was assumed to move along the observed track without change in intensity or structure. As the pre-storm ocean currents were not measured, the ocean was assumed to be initially at rest. The simulated SST decrease was rather close to the observed values. Strong upwelling in the center of the loop-shaped track after 180 h simulation had about the observed amplitude (Fig. 5.16). The differences between the calculated and observed mixed-layer depths are probably because of the layer shallowing due to net heating that probably took place after the typhoon passage, which was not simulated by the model physics. A cyclonic eddy was simulated in the upper ocean due to geostrophic adjustment of the dome-shaped upwelling generated by the typhoon and the current field. This was consistent with observations of the current velocities made after the Typhoon Virginia passage (Ivanov et al. 1980).

Cooper and Thompson (1989a) studied the ocean response to three hurricanes in the Gulf of Mexico using a four-layer model based on the full system of the layer model equations (5.28)-(5.37), and also including bottom topography. The temperature and salinity equations were consolidated into a single equation for the density balance. The objective was to simulate the mixed-layer current response generated by hurricanes on the outer continental shelf and slope. The simulated mixed-layer currents in response to Hurricanes Frederic and Eloise agreed rather well with the observed values. Poor agreement in the thermocline and the bottom layers was probably caused by the poor

Fig. 5.16 Vertical cross-sections of (a) observed and (b) simulated temperatures along 22°N for Typhoon Virginia. The model simulation is for 180 h after the beginning of typhoon action (Ginis and Dikinov 1989).
resolution of the bottom topography. Some of the discrepancies may also be due to the omission of the initial currents. A model sensitivity study (Cooper and Thomson 1989b) indicated that the baroclinic component of the ocean current exceeded the barotropic component even for water depths of O(100 m). The current response was sensitive to the parameterization of entrainment. Comparison with data from Hurricane Frederic indicated that equation (5.42) gave more realistic mixed-layer currents than (5.41).

Ginis et al. (1992) compared simulations of an eight-layer, reduced-gravity model with the field observations of the ocean response to Hurricane Norbert. The model was based on the full system of equations (5.28)-(5.38) with the mixing parameterization in (5.43)-(5.45). The numerical simulations were started far beyond the survey region depicted in Fig. 5.7a and stopped at 0000 UTC 24 September 1984 when the survey took place. The storm was moved along the Norbert best track. The axisymmetric wind stress profile was obtained based on flight-level data at 1500 m and remained the same during the integration. For real ocean simulations, the most difficult problem is specifying initial conditions, which requires knowledge of the pre-storm three-dimensional temperature and current fields that are very difficult to obtain. Since the Norbert data set did not include a pre-storm survey, it was necessary to make assumptions regarding the ocean initial conditions. Fortunately, the Norbert survey region is not dynamically very active (Sanford et al. 1987) and with probably small loss of realism, the impact of the background thermodynamic fields on the hurricane-generated ocean response could be neglected. Therefore, the ocean was assumed to be initially homogeneous and quiescent. The SST and the mixed-layer depth were set to 28°C and 30 m, respectively, with a typical temperature profile in the thermocline for that region. The simulations (Fig. 5.17) give roughly comparable values of the mixed-layer velocities and SST decreases at AXCP positions (Sanford et al. 1987), but also suggest that larger cooling might have occurred south of the survey region, with a decrease of 4.5°C in the area where the storm made its turn of near 90°. No observations were taken in this region. Price (1994) calculated a very similar SST distribution for the Norbert case using a level model.

The most overt feature of the predicted ocean response is a rightward bias in the current, temperature, and mixed-layer depth fields. This is in accord with observations and other models discussed above. Since Norbert moved slowly in the survey region (U_h ~ 3 m s^{-1}), upwelling was generated rather close to the storm center (Fig. 5.17f). The maximum value of the pressure-driven circulation in the third model layer (at depths ~ 75-125 m) was 0.37 m s^{-1} and was located about 110 km behind the hurricane center (Fig. 5.17d). The upwelling and thermocline currents might have significantly influenced the mixed-layer depth (Fig. 5.17e) and SST decrease in the rear part of the hurricane (Fig. 5.17b). It should be noted that the maximum mixed layer depth calculated in the model (about 70 m) exceeded the observed values of 40-45 m. This is an inherent model failure mainly related to the poor vertical resolution of a layer model. As indicated in the previous section, the layered model (5.28)-(5.38) employs a mixed layer formulation that confines all turbulent mixing to a surface mixed-layer and thus develops a sharp jump in density and current across the base of that layer. The observed vertical profiles in the Norbert case have a continuous and smooth variation of density and current across the mixed-layer base. This variation occurs over the transition layer that is simulated by a discontinuity surface in the layer model. Therefore, a direct comparison of observed mixed-layer depth, usually based on a vertical gradient criteria, with the simulated mixed-layer depth will require significant interpretation of the model results.

5.4.3 Simulations of the ocean response in level models

Chang (1985) tested an axisymmetric level model that included a free surface and flat bottom and simulated the response to a surface wind stress similar to Hurricane Eloise. The vertical resolution included 51 grid points between the surface and 1500 m, with highest resolution of 5 m near the surface. The EB-10 sounding on September 1975 was used as an initial temperature profile. After the hurricane forcing was applied for 48 h, the
Fig. 5.17 (a) Track and wind stress for Hurricane Norbert case with translation speeds \( \text{m s}^{-1} \) indicated in the circles. Predicted (b) SST deviations \( \text{°C} \) and observed values (rectangles); (c) mixed-layer velocities and observed velocities (bold arrows); (d) thermocline (layer 3) velocities; (e) mixed-layer depth; (f) the 16°C isotherm displacement from initial depth (Ginis et al. 1992).
computations were continued without forcing for another 96 h to study the post-storm adjustment. The response of the upper ocean was qualitatively similar to layer model studies with a stationary storm. For example, the mixed layer response simulated by this model agreed well with a two-layer, reduced-gravity model. However, some new aspects of the ocean response were simulated. The ocean current response extended to a great depth, which cannot be simulated with a reduced-gravity model. The maximum barotropic tangential velocity reached 19 cm s\(^{-1}\) at 48 h and constituted a major part of the deep response. The sea surface height decreased to 57 cm near the storm center. During the post-storm period, near-inertial oscillations with large amplitudes extended over much of the ocean depth as the mass field and momentum field adjusted. The spindown of the barotropic part of the response was very slow, with the sea-surface depression being reduced to 26.7 cm after six days.

Shay et al. (1990) used the level model of Chang (1985) for a moving hurricane. The model was initialized with a salinity-temperature depth profile acquired in the northern Gulf of Mexico. The total ocean depth was 610 m with a flat bottom. The idealized forcing was patterned after the parameters observed in Hurricane Frederic and was moved with a speed of 5.5 m s\(^{-1}\) westward. The simulated upper ocean response was very similar to the previous studies using reduced-gravity model. Maximum mixed-layer velocities of about 140 cm s\(^{-1}\) were to the right of the storm. However, the thermocline baroclinic response is quite different. The simulated thermocline currents in the cross-track direction are 180° out of phase with the mixed layer currents (Fig. 5.18a), which indicates that the first mode prevails in the thermocline baroclinic response for this shallow ocean simulation. This is in contrast to the deep-ocean response in Fig. 5.15, which has a lag between the mixed layer and thermocline currents. The current reversal increased the vertical shear, which enhanced mixing of heat and momentum. As a result, the depth of the current reversal increased to 200 m after one inertial period (Fig. 5.18a). There is also an evidence of energy propagation downward. Maximum thermocline velocities increased to 20-30 cm s\(^{-1}\) between 400-600 m after about 5 IP. The mixed layer temperature decreased by 1.0-1.2 °C at x=R\(_m\) following storm passage (Fig. 5.18b). Positive temperature deviations below

![Fig. 5.18](image-url)

Fig. 5.18 (a) Cross-track component of current (cm s\(^{-1}\)) from Shay et al. (1990) numerical model at x=4R\(_m\) with positive (negative) components depicted as solid (dashed) contours at an interval of 10 cm s\(^{-1}\). (b) Temperature increases (decreases) are indicated by solid (dashed) contours with an interval of 0.2°C. The abscissa is scaled by the near-inertial wavelength (580 km) and depth (U) is WKBJ-stretched.
the mixed layer are mainly due to penetration of the surface warm water downward by turbulent mixing. Although the maximum positive temperature anomalies are simulated at the top of the thermocline where the gradient of temperature is the largest, the magnitude varies in time due to the near-inertial pumping induced by divergence and convergence of upper ocean currents. The tropical cyclone also generated a significant barotropic response in this shallow water simulation. At the sea surface, a trough in the along-track direction with the maximum amplitude of about 20 cm was superposed with near-inertial oscillations of about 4-6 cm. The barotropic current calculated from the gradient of the sea-surface elevation indicated the along-track velocity of 12-14 cm s\(^{-1}\) and the cross-track velocity of 4-8 cm s\(^{-1}\) of the near-inertial wavelength. Based on this result, Shay et al. concluded that a barotropic current response occurs in the near-inertial wave band as well as in the baroclinic response.

Price et al. (1994) simulated the ocean response to three hurricanes (Norbert and Josephine during 1984, and Gloria during 1985) using a reduced-gravity, level model. The gradient pressure was assumed to be zero below the 1000 m. The vertical grid spacing was 10 m within the upper 150 m and increased to 50-100 m at depths down to 1000 m. The only subgrid scale process included in the model was the upper-ocean vertical mixing that was parameterized by (5.46)-(5.48). The wind stress patterns were calculated based on observations obtained during research flights. For all the cases considered, the ocean response was dominated by near-inertial currents. Detailed comparisons of the model simulations with the AXCP observations indicated reasonably good agreement, especially in the regions of strong current response (Fig. 5.19). Because of the slow translation speed of Norbert in the survey area, the dynamical pressure coupling between the mixed layer and the thermocline was rather strong. This resulted in a thermocline-depth current of 0.3 m s\(^{-1}\) under the rear half of the hurricane. Ginis et al. (1992) obtained similar results using a layered model (Fig. 5.17d).

The observed and simulated temperature profiles (Fig. 5.19) have only a modest increase of the mixed-layer depth. By comparing the temperature profiles from AXCP data taken ahead of Norbert (AXCP N2, Fig. 5.7) and the averaged temperature profile from all AXCPs behind the hurricane eye, Price et al. found that much of the vertical mixing occurred within the upper thermocline rather than solely within a deepening mixed layer. The initial mixed-layer depth was about 40-45 m, but the vertical mixing reached depths of nearly 80 m. Thus, the modest increase of the mixed layer depth during hurricane forcing may be an inadequate measure of the depth of vertical mixing.

A tropical cyclone transfers energy to the ocean almost entirely through the wind stress. Therefore, successful simulations of the ocean response require a proper parameterization of this important process. The usual practice in numerical modeling is to use the conventional bulk formula for the wind stress calculation. For hurricane conditions, gravity waves on the ocean surface may possibly modify the momentum transfer from the atmospheric boundary layer to the ocean mixed layer (see Chapter 5.6). Price et al. (1994) tested the wind stress calculated from a bulk formula using the drag coefficient of Large and Pond (1981) and flight-level winds. The simulated transport in the upper-ocean layer of 80 m depth was compared with the transport calculated from the AXCP. Such a test is possible because the simulated transport is directly proportional to the wind stress amplitude and is almost independent of the model-specific parameterization of vertical mixing. The comparison was made only for the currents greater than 0.7 m s\(^{-1}\) to isolate the storm-driven response from pre-existing currents. Price et al. concluded that for the purpose of modeling the ocean current response to a hurricane, the wind stress can be estimated from aircraft-measured winds and the bulk formula within an accuracy of about 20%.

In summary, the model simulations described in this section provide a valuable framework for interpreting the widely-spaced observations of the ocean response to tropical cyclones. The continuous model fields reveal the three-dimensional temperature and current structure that are difficult to obtain from rather sparse observational data.
Fig. 5.19 Current and temperature profiles from Norbert AXCP N13 (top left) and N14 (bottom left) (the locations of the profiles are in Fig. 5.6b) and from the Price et al. (1994) simulation at the same locations (right). At these locations, which were behind and to the right of the eye, an appreciable thermocline-depth current (depth below 70 m) flows toward the low pressure anomaly centered about 100 km behind the eye.

They also indicate the dynamical relationships among variables during and after the cyclone passage that are hard to infer from the field data alone. Although some simulations have fairly good reproductions of the observed magnitudes and positions of the temperature and current features generated by a tropical cyclone, most of the simulations have treated highly idealized initial conditions and forcing distributions. Therefore, much work remains to be done to convert the research models to real-data operational models. We envision two major directions in the ocean model development. First, a numerical model should include bottom topography and coastal geography to simulate realistically the ocean response during all stages of hurricane life cycle: over open deep ocean, on the outer continental shelf, and in shallow-water, coastal regions. Second, a model should be capable of incorporating a realistic three-dimensional structure of the initial oceanic fields. Both tasks are especially difficult to accomplish due to the very high horizontal and vertical resolutions required to simulate properly the physical processes outlined in this section.
One of the possible directions for the ocean model development may be based on a multi-nested grid system with higher resolutions in the vicinity of the hurricane track.

5.5 SIMULATIONS OF TROPICAL CYCLONE-OCEAN INTERACTION

5.5.1 Early studies with axisymmetric tropical cyclone models

In early coupled numerical simulations of tropical cyclone-ocean interaction by Chang and Anthes (1979) and Sutyrin and Khain (1979), axisymmetric tropical cyclone models were coupled with two-layer, reduced-gravity ocean models. Early in these integrations, SST decreases did not have a large effect on the tropical cyclone evolution or structure. In the Chang and Anthes model (1979), the minimum sea-level pressure rose by only 2 mb in one day in response to a SST decrease of almost 3°C. In Sutyrin and Khain (1979), the axisymmetric storm was moved over the ocean with a speed of 5 m s⁻¹. A noticeable feedback occurred on the second day of the integration, during which the minimum sea-level pressure increased by 6 mb and the maximum velocity decreased by 15%. Sutyrin and Khain (1984) studied the sensitivity to the cyclone translation speed and the initial mixed-layer depth. As expected, the storm weakening was larger (i.e., 5-10 mb) for slower moving storms and for shallower mixed-layer depths. When the ocean coupling was included from the beginning of the integration, development of the tropical cyclone was slower, and the maximum intensity was substantially weaker (10-15 mb).

5.5.2 Three-dimensional studies of tropical cyclone-ocean interaction

Ginis et al. (1989) developed a three-dimensional coupled model with a five-level tropical cyclone model (Khain 1988) and a three-layer ocean model (Ginis and Dikinov 1989). The tropical cyclone model was based on the full system of hydrothermodynamic equations on the beta plane. Condensational heating, moisture, and precipitation were calculated at the grid-resolvable scales (Rosenthal 1978; Khain 1983). The surface heat fluxes were calculated using the Deardorff (1972) parameterization. Since the size of the computational domain was small, the model integrations did not include an environmental flow, and the storm motion was due only to the beta effect (see Chapter 4.3). Cooling of the upper ocean by 1.5°C under the tropical cyclone led not only to a decrease in intensity (by 6 mb), but also to a more rapid northward displacement of the cyclone.

Khain and Ginis (1991) also used the Ginis et al. (1989) coupled model and crudely included the effect of a basic flow by moving the atmospheric grid over the ocean grid. Grid resolution in both the tropical cyclone and ocean models was 40 km. An atmospheric model integration to 24 h with constant SST resulted in a tropical cyclone with a minimum pressure of about 1000 mb, and the interaction with the ocean was then introduced. The initial thermodynamic structure of the ocean was assumed horizontally homogeneous and motionless. The atmospheric grid was moved westward, eastward, or northward above the ocean grid with prescribed velocities. All coupled model experiments (AO) were repeated with the uncoupled model (A) that had fixed SST. Some specified parameters and some results of the numerical experiments are summarized in Table 5.2.

The main conclusion from these experiments (Fig. 5.20) is that both the intensity (i.e., 5-7 mb) and motion of the tropical cyclone were affected by the cooling of the upper ocean (i.e., 3.5-6.1°C). In the fixed SST experiments A1, A3, and A4 with westward grid displacements of 0, 5, and 8 m s⁻¹, respectively, the tropical cyclones were more intense and were displaced farther to the north than in the corresponding coupled experiments AO1, AO3 and AO4. However, the more intense storms for the eastward grid movement and the fixed SST experiments A6 and A7 were displaced to the north less than in the corresponding coupled experiments AO6 and AO7. The more intense storm in experiment AO7 was displaced northward slightly more than the cyclone in experiment AO6. In experiments A6 and A7, the opposite trend is observed. It is important to note one more feature. Comparing just the coupled model experiments, eastward-moving
Table 5.2 Specification of parameters and results of a series of experiments with the coupled hurricane-ocean (AO) and non-coupled (A) models. $U$ is the imposed translation speed and direction of the atmosphere grid over the ocean (Khain and Ginis 1991).

<table>
<thead>
<tr>
<th>Experiments</th>
<th>$U$ (m s$^{-1}$)</th>
<th>Minimum surface pressure (mb)</th>
<th>Maximum SST anomaly ($^\circ$C)</th>
<th>Maximum two-day displacement to the north (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AO1</td>
<td>0</td>
<td>983.5</td>
<td>-5.9</td>
<td>120</td>
</tr>
<tr>
<td>A1</td>
<td>0</td>
<td>977.0</td>
<td>0</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>westward</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AO2</td>
<td>2</td>
<td>982.0</td>
<td>-6.0</td>
<td>95</td>
</tr>
<tr>
<td>AO3</td>
<td>5</td>
<td>981.6</td>
<td>-4.5</td>
<td>115</td>
</tr>
<tr>
<td>A3</td>
<td>5</td>
<td>975.0</td>
<td>0</td>
<td>165</td>
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<td>AO4</td>
<td>8</td>
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<td>-3.5</td>
<td>110</td>
</tr>
<tr>
<td>A4</td>
<td>8</td>
<td>974.7</td>
<td>0</td>
<td>173</td>
</tr>
<tr>
<td>AO5</td>
<td>2, for $t \leq 45.5$ h</td>
<td>977.5</td>
<td>-4.6</td>
<td>135</td>
</tr>
<tr>
<td></td>
<td>8, for $t &gt; 45.5$ h</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>eastward</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AO6</td>
<td>2</td>
<td>981.4</td>
<td>-6.1</td>
<td>163</td>
</tr>
<tr>
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<td>2</td>
<td>975.0</td>
<td>0</td>
<td>140</td>
</tr>
<tr>
<td>AO7</td>
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<td>175</td>
</tr>
<tr>
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<td>974.5</td>
<td>0</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>northward</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AO8</td>
<td>5</td>
<td>979.0</td>
<td>-4.4</td>
<td>150</td>
</tr>
<tr>
<td>A8</td>
<td>5</td>
<td>972.0</td>
<td>0</td>
<td>200</td>
</tr>
</tbody>
</table>

Fig. 5.20 Two-day poleward displacement and maximum intensity (minimum pressure) in Khain and Ginis (1991) coupled hurricane-ocean experiments (see description in Table 5.2). The right half is for cases with the atmospheric grid displaced eastward and the left half is for cases with westward displacements.
storms were displaced farther to the north compared with the westward-moving storms. In the fixed SST experiments, the westward-moving ones were displaced more.

These differences in the relationships between northward displacement and tropical cyclone intensity for eastward- and westward-moving storms are connected with an asymmetry in the precipitation patterns, which is in turn related to the asymmetry of the heat and moisture fluxes from the sea surface relative to the storm center. Khain and Ginis have suggested the following explanation for the connection between the anomalies of heat and moisture fluxes from the sea surface and anomalies in the precipitation pattern. When the SST is fixed, an asymmetry of the fluxes across the storm track occurs due to larger wind velocities on the right side of the track. In the coupled-ocean experiments, this asymmetry is modified since a region of colder water exists to the rear and right of the storm center (Fig. 5.21). After an air parcel passes over the positive surface heat flux anomaly region in Fig. 5.21b, it moves around the storm center through an angle $\alpha$ before it is lifted to the condensation level and then ascends more vertically to release latent heat. This cyclonic rotation angle depends on the distribution of the tangential and vertical velocities and, therefore, on the tropical cyclone intensity or stage of development. The angle was evaluated by following the motion of some air particles from the tropical cyclone boundary layer. In the coupled model (Fig. 5.21a), this angle was about $90-130^\circ$, with the largest values for the weakening stage. In the uncoupled model (Fig. 5.21b), this angle was a little less ($80-120^\circ$). These differences are small compared to the difference between the angular differences of the positive surface heat flux anomaly regions between the coupled and fixed SST experiments. The surface flux anomaly thus is the important factor in the gradient of the heating anomalies. Owing only to the gradient of heating, the tropical cyclone in the coupled (fixed SST) model will tend to propagate to the left (right) of the large-scale steering flow. Adding this baroclinic propagation effect to the beta-effect propagation may account for the differences in the northward displacements in Fig. 5.20.

![Fig. 5.21 Schematic of the asymmetry of energy supply from the ocean and the asymmetry in condensation heat release for (a) a TC interacting with the ocean, and (b) for a TC with fixed SST (Khain and Ginis 1991). The anomalies of surface heat fluxes and of condensation heat are relative to a large-scale steering flow from left to right in both cases, and the motion contributions owing to condensation heat release gradients are indicated by the thin arrows.](image-url)
Fig. 5.22. Mean 24-h storm displacements in the North Atlantic (dashed) and western North Pacific (solid) for (a) tropical depressions (D), tropical storms (S), and typhoons or hurricanes (T). Eastward-moving (right side) cyclones are grouped separately from westward-moving (left side) cyclones (Dong 1988).

These results of coupled model simulations seem to be consistent with observational analyses by Dong (1988), who examined mean 24-h displacements of a large set of westward- and eastward-moving storms in the western North Pacific (10,386 cases) and North Atlantic basins (20,622 cases). His results (Fig. 5.22) show that more intense tropical cyclones usually move to the north more rapidly and that eastward-moving storms are displaced northward more than those moving westwards. Whereas the prediction of the coupled model agrees with these observations, the motion tendency from the uncoupled model would give the opposite results.

One of the restrictions of the coupled model used in Khain and Ginis (1991) is the crude consideration of the environmental flow. In a coupled tropical cyclone-ocean model developed at GFDL (Ginis et al. 1992; Bender et al. 1993), a tropical cyclone could move over the ocean in prescribed easterly and westerly zonal atmospheric flows. The coupled model includes the GFDL tropical cyclone prediction model with an eight-layer ocean model that had been separately tested for the Hurricane Norbert case (see Chapter 5.4). The GFDL tropical cyclone model has a triply-nested, movable mesh system with the finest mesh resolution of 1/6° lat./long. The ocean model has a uniform grid spacing of the same resolution. In all coupled model experiments, the ocean was initially assumed to be horizontally homogeneous and quiescent, with a vertical thermal structure profile provided by Black (1983). The environmental conditions for Hurricane Gloria on 1200 UTC 22 September 1985 served as the initial mass and moisture fields for the tropical cyclone model. The symmetric vortex was generated from time integration of an axisymmetric version of the model as described in Kurihara et al. (1993) and inserted in prescribed easterly and westerly flows of 7.5, 5.0, and 2.5 m s⁻¹. All of the integrations were extended to 72 h with the tropical cyclone-ocean interaction included from the beginning.

The SST anomalies simulated by the GFDL coupled model are shown in Fig. 5.23 for a 5 m s⁻¹ easterly basic flow. The regions of maximum SST decrease of about 3.5°C are located about 400 km behind the storm center and about 100 km to the right of the track. Directly underneath the storm center, the SST decreases are 1.5°C. Although this SST pattern is consistent with observations discussed in Chapter 5.2, the SST decreases appear to be larger because of the strong hurricane forcing. The maximum surface wind simulated by the tropical cyclone model was about 50 m s⁻¹ (Fig. 5.23). These SST decreases produced significant reductions in the total surface heat flux (latent plus sensible) into the hurricane. In the 5 m s⁻¹ basic flow case (Fig. 5.24), the maximum total heat flux averaged for the entire integration period decreased from 1.25 kw m⁻² in the fixed SST model to about 0.9 kw m⁻² in the coupled model. Most of the decrease in the total heat flux in these
Fig. 5.23 SST anomalies (°C) and near-surface wind vectors at 72 h for the Bender et al. (1993) coupled experiments with 5 m s⁻¹ easterly basic flow. Only the region of the innermost nest is shown, with the tick marks at 1° lat./long. intervals. The region with SST decreases larger than 2°C is shaded. The storm center at 72 h is indicated by the hurricane symbol and the storm track is indicated by the thick solid line.

Fig. 5.24 As in Fig. 5.23, except for total surface heat flux (kw/m²) (positive value directed upward into the atmosphere) averaged for the entire 72 h of the integration and computed relative to the moving storm for (a) the coupled hurricane ocean model and (b) with fixed SST (Bender et al. 1993).
two experiments was in the latent heat flux. In the coupled model, the pronounced asymmetry in the total surface heat flux distribution is primarily over the cold wake as in the schematic in Fig. 5.21a. In accordance with the reduction of the total heat flux, the total storm precipitation accumulated relative to the moving storm significantly decreased for all sets of coupled experiments. The largest decrease was in the eyewall region of the storm quadrant in which the largest decrease in surface heat flux occurred. With the 5 m s\(^{-1}\) easterly basic flow, the accumulated precipitation was reduced between 10% to 20% (maximum decrease of about 60 cm over 72 h) on the eastern side of the storm center over the cold wake. In contrast, only a 5% to 10% (maximum decrease of about 20 cm) occurred on the western side where the reduction of heat fluxes due to the ocean coupling was much smaller.

Decreased surface heat flux owing to the ocean coupling is reflected in the horizontal distributions of the near-surface equivalent potential temperature (Fig. 5.25). Over the cold wake underneath and to the east of the storm center, the equivalent potential temperature is decreased by about 2\(^\circ\) to 6\(^\circ\)C (Fig. 5.25c) relative to the fixed SST experiment. Although the area of the cold wake was rather narrow (Fig. 5.23), the effect on the equivalent potential temperature is spread over a large region behind and to the right of the storm center. Smaller decreases in the equivalent potential temperature are found in advance and to the left of the center, and these regions may contribute more heat energy to the storm. However, the difference in the equivalent potential temperature in the lowest 500 m between the storm center and an azimuthal average on the outer storm periphery (-450 km) decreased from 16\(^\circ\) to 12\(^\circ\)C owing to the ocean coupling.

A summary of the Bender et al. (1993) coupled and fixed SST simulations with various basic flows is given in Table 5.3. The storms embedded in the westerly basic flows moved slower that those with corresponding easterly flow since the basic flow in these cases was oriented in a direction almost opposite to the flow owing to the beta effect. This resulted in larger sea-surface temperature decreases with the storms moving in the westerly flow and a greater decrease in intensity for these cases. A strong correlation was found between the maximum SST decrease and the changes in storm intensity for both the eastward- and westward-moving storms. The faster-moving storms produced a progressively smaller SST decrease and a smaller reduction in tropical cyclone intensity as measured by both the minimum sea-level pressure and maximum surface winds.

The largest impact on the storm track occurred with slow moving storms. In the experiment with 2.5 m s\(^{-1}\) basic flow (Fig. 5.26), the storm in the coupled model gradually turned more to the north of the storm in the fixed SST experiment. Bender et al. (1993) suggest this track deviation is related to a systematic decrease in the azimuthally averaged tangential flow at all radii in the coupled model (not shown). Fiorino and Elsberry (1989) found that differences of about 2 m s\(^{-1}\) in the tangential wind profiles at 300 - 700 km radius of an idealized vortex resulted in similar changes in the storm track in a non-divergent, barotropic model with no basic current (see Chapter 4.3). For the simulation with the 7.5 m s\(^{-1}\) basic flow, no significant track deflections occurred (Fig. 5.26) except for a slightly faster movement in the coupled model. This is consistent with the baroclinic mechanism in Fig. 5.21a since the maximum heat release was found ahead of the tropical cyclone in the coupled experiment.

In summary, these recent coupled tropical cyclone-ocean simulations suggest some potentially important effects that ocean interaction may have on tropical cyclones. For slowly moving tropical cyclones, the intensity of the tropical cyclone may be affected by its wake of lower SST. However, these simulations have not considered other environmental factors that are believed to affect the intensity of tropical cyclones (see Chapter 3). Clearly, some environmental factors inhibit the majority of tropical cyclones from reaching their maximum potential intensity (e.g., Emanuel 1988). Tuleya and Kurihara (1982) numerically simulated types of environmental conditions that may significantly alter the
Fig. 5.25 Near-surface ($\sigma=0.98$) equivalent potential temperature (K) after 72 h of the 5 m s$^{-1}$ easterly basic flow experiment of Bender et al. (1993) (a) with and (b) without ocean coupling, and (c) decrease of the equivalent potential temperature in the coupled experiment relative to the non-coupled experiment. The contour interval is 2 K, with values less than 352° and 350° in (a) and (b) and values greater than 0° and 2° in (c) indicated by lighter and thicker shading, respectively.

Fig. 5.26 Tropical cyclone tracks in the Bender et al. (1993) simulations with (a) 2.5 m s$^{-1}$ and (b) 7.5 m s$^{-1}$ easterly basic flow. Positions are given each 6 h for the coupled (circles) and non-coupled (triangles) models.
sensitivity of tropical storm genesis to the SST. These recent coupled model studies indicate that the distribution of the energy supply to the storm will be significantly modified in cases with large SST decreases, which will reduce the maximum intensity. Whereas the above numerical simulations are for open-ocean conditions, many of the observations have been made in the coastal regions in which the oceanographic processes are more complicated. Comparisons of these observed cases with improved numerical ocean models are necessary to clarify the role of the tropical cyclone-ocean interaction in the real atmosphere and ocean.

Emerging technological developments in aircraft-based instrumentation have recently provided high-quality measurements of atmospheric and oceanic parameters in the vicinity of a tropical storm in real time (Chapter 5.2). By skillfully assimilating these datasets into coupled air-sea models, they have a large potential to improve substantially the tropical cyclone track and intensity predictions. While various data assimilation techniques are currently used for tropical cyclone forecasting, future research will be necessary to develop appropriate schemes for assimilation of ocean data into the primitive-equation models.

5.6 SURFACE SEA STATE

The heat, moisture, and momentum exchanges between a tropical cyclone and the ocean involve microscale physical processes near the sea surface that include spray production and advection, the characteristics of the interfacial sublayers, and the character of the surface sea state. Existing parameterizations of these exchanges have yet to account for all important mechanisms that can influence both media. For example, the momentum flux calculations are usually based on the Charnock (1955) relation for the roughness
length, which appears to apply only in the case of a fully developed surface wave spectrum with no swell. These conditions can hardly be applied for the wave fields having the space and time scales generated by mature tropical cyclones. Surface wavelengths in the vicinity of a tropical cyclone range from a few centimeters for capillaries to hundreds of meters for the longest ocean swells. In this section, the available observational data on the structure of the wave field in a tropical cyclone are described that may play an important role in the interaction between a tropical cyclone and the ocean.

5.6.1 Observational studies

Observations of the sea surface wavefield generated by a tropical cyclone are very limited. Even under normal atmospheric conditions, observing the spatial structure of the sea surface is both technically difficult and expensive. Typical measurements of waves in the form of time series of surface elevations at a point do not give spatial information about the wavefield, which is of primary interest here. Such observations may not be very accurate during a tropical cyclone passage due to the harsh wind conditions.

Although remote sensing techniques seem to offer the best alternative to observe the spatial distribution of wave parameters, only a few of the commonly used remote techniques can be applied for tropical cyclones. The most direct way of obtaining information about the sea surface is from photographs, including stereography, whitecap measurements, and slope spectra from single photographs. Of these, stereophotography potentially gives the most information about the spatial structure of the surface. The data obtained from a pair of stereophotographs is an elevation map of the sea surface. The biggest problem with the use of stereophotography is that the two cameras need to be synchronized very precisely or the motion of the sea surface will lead to a blurred stereo-image. This is complicated because the cameras are usually on two separate aircraft. Holthuijsen (1983) gives a full account of the difficulties involved. Other photographic observations, such as measurements of light reflected from the sea surface or measurements of whitecap coverage also have limited use in high wind conditions. The first method does not apply with significant whitecapping, and the second one gives little

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Fig. 5.27 Surface Contour Radar wave spectrum (m$^2$ Hz$^{-1}$ deg$^{-1}$) in region of Hurricane Debby showing both the real spectral lobes and the ambiguous lobes (crosses) (Walsh et al. 1985).
information on the spatial statistics of wave breaking since the whitecap coverage crucially depends on the persistence of foam, which is not readily determined.

Other means of obtaining spatial wave information at the sea surface are direct radar measurements from aircraft with Surface Contour Radar (SCR) and side-looking air-borne radars (SLAR). Walsh et al. (1985) successfully used a SCR during Hurricane Debby in 1982 to obtain a directional wave spectrum (Fig. 5.27) approximately 240 km from the center. The spectra of the surface waves consist of a low-frequency swell and a higher frequency wind sea that propagates in a different direction. The swell peak spectral density is at 0.09 Hz and the spectral peak of the wind sea is at 0.12 Hz. The sea (swell) spectral lobe turns clockwise (counterclockwise) through 30° in the direction of propagation as the frequency increases. Walsh et al. estimated that the sea and swell were generated in regions 100 and 140 km from the eye of the hurricane, respectively. This study has demonstrated that SCR is capable of providing measurements of sea-surface directional spectra with high resolution.

Black et al. (1988a) described the sea-surface wavefield and wave-induced motions in Hurricanes Norbert and Josephine in 1984 using AXCP measurements and SLAR data. The presence of energetic surface waves was evident in all AXCP velocity profiles acquired in the vicinity of the hurricanes. The near-surface velocity changes in the wave-induced current were more than 100 cm s\(^{-1}\) over the upper few tens of meters (exponential decay with depth). Black et al. removed the orbital velocity associated with large waves (swell) from the AXCP data using a three-layer, least-squares model developed by Sanford et al. (1987). The wave-induced motion had a period of about 10 s, which according to the linear dispersion relation \( \omega^2/g \) corresponds to a wavelength of 156 m. SLAR measurements gave a larger estimated wavelength of 200 m. This may be an indication of nonlinear processes involved in the dynamics of the hurricane-induced surface waves. The maximum surface currents induced by the swell in Hurricane Norbert (Fig. 5.28) were 80 cm s\(^{-1}\) and were located in the right quadrant of the storm. These observations thus demonstrated that the highest waves and longest fetches are to the right of the storm track.

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Fig. 5.28 Streamlines indicating 200 m swell propagation direction near Hurricane Norbert. Surface wave-induced orbital velocity (cm s\(^{-1}\)) is contoured with dashed lines. Triangles indicate swell directions obtained from side-looking airborne radar images (Black et al. 1988b).
The largest surface wave height in Norbert estimated by Sanford et al. (1987) based on AXCP data was 7.1 m. Black et al. (1989) pointed out that the pattern of swell propagation was nearly identical to the wind stress driven mixed-layer current. They suggested that this wave-induced motion may modulate the vertical current shears, and hence the vertical turbulent mixing in the upper ocean. Shay et al. (1994) used an airborne scanning radar altimeter (SRA) and obtained AXCP observations simultaneously to measure the wave field and currents in the vicinity of the Gulf Stream during the Surface Dynamics Experiment in 1991. The SRA directly measured the sea surface topography, which was transformed into directional wave spectra at 5-6 km intervals along the flight tracks. Shay et al. found good agreement between the two independent measurements of the orbital velocity amplitudes, and the wave propagation direction inferred from the AXCP-derived orbital velocities was the same as from the SRA. The mean wave directions were highly correlated (0.87) and differed by only about 5°. These results demonstrate the potential for using airborne technology to make surface wave measurements during tropical cyclones.

5.6.2 Sea state effects on the air-sea momentum transfer

The importance of the air-sea exchange in the tropical cyclone and ocean modelling can hardly be overestimated. Because of a growing interest in the simulations of air-sea interaction by means of coupled atmosphere-ocean models, much attention has been recently devoted to improving the parameterization of the air-sea fluxes. In this section, recent theoretical and numerical studies of ocean waves and the role of the sea state in the transfer of momentum from air to ocean will be discussed. Such wave models have yet to be implemented in coupled models of tropical cyclones and the ocean. Implementation of such wave-dependent momentum fluxes may greatly improve the simulations of the hurricane evolution and ocean response, including storm surge forecasting to be discussed in Chapter 5.7.

The wind stress \( \tau \) on the sea surface depends on the wind speed and the roughness of the water \( z_0 \), which depends on the waves that are present. Charnock (1955) derived from dimensional arguments an implicit relation between the roughness and the wind \( z_0 = \alpha u_k^2/g \), where \( u_k = (\tau/\rho)^{1/2} \) is the friction velocity. A unique relationship between the drag coefficient \( C_D \) and \( z_0 \) exists that leads to a roughly linear relation between \( C_D \) and the wind. This type of drag coefficient is commonly used in tropical cyclone models. Other variables enter the problem if the waves are not fully determined by the wind. If the growth of the waves is limited by fetch, time, or depth, these quantities will influence the interaction between air and sea. Recent experimental evidence demonstrates the wave-state dependence of the drag coefficient. Geemeart (1990) showed that these drag relations depend on the depth of the sea in which they were measured. Janssen (1992) found that the roughness depends explicitly on the ratio \( c_p/u_k \), where \( c_p \) is the phase speed of the peak of the spectrum.

The generation of surface waves is due initially to turbulent fluctuations that perturb the sea surface and resonate with a length scale that becomes unstable and grows into a waveform. This energy input may then be transformed to other wavelengths via nonlinear wave-wave interactions. Prediction models must then predict a wave spectrum \( F(f, \theta) \), where \( f \) is the frequency of a wave component, and \( \theta \) is the direction. The energy balance equation for waves is

\[
\frac{\partial F}{\partial t} + \nabla \cdot (c_g F) = S_{in} + S_{nl} + S_{ds},
\]

where \( c_g \) is the wave group velocity vector. The first term on the right side of (5.51) represents the atmospheric input (wave generation), the second term represents the rate of nonlinear spectral transfer (wave-wave interaction), and the last term describes the dissipation of wave energy due to breaking, turbulence, and viscosity. For example, the third-generation WAM model (WAMDI Group 1988) explicitly includes generation, dissipation, and nonlinear wave-wave interactions. The WAM model was successfully
Table 5.4 Summary of WAM wave model (WAMDI Group 1988) hindcasts versus measured peak significant wave height (m) for three Gulf of Mexico hurricanes.

<table>
<thead>
<tr>
<th>Storm</th>
<th>Site</th>
<th>Measured</th>
<th>Hindcast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camille</td>
<td>ODGPSTN1</td>
<td>13.6</td>
<td>13.5</td>
</tr>
<tr>
<td></td>
<td>ODGPSTN2</td>
<td>7.9</td>
<td>9.0</td>
</tr>
<tr>
<td>Anita</td>
<td>EB04</td>
<td>5.4</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>EB71</td>
<td>6.6</td>
<td>7.7</td>
</tr>
<tr>
<td></td>
<td>El135</td>
<td>6.0</td>
<td>5.1</td>
</tr>
<tr>
<td>Frederic</td>
<td>42003</td>
<td>8.9</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>Cognac</td>
<td>9.5</td>
<td>8.1</td>
</tr>
</tbody>
</table>

tested against wave measurements obtained during the passage of three hurricanes in the Gulf of Mexico: Camille (1969), Anita (1977), and Frederic (1979) (WAMDI Group 1988). In spite of the rather crude representations of the hurricane surface wind fields, good agreement was found between modeled and measured wave heights (Table 5.4) and estimated frequency spectra. These comparisons indicated that the spectral shape and peak frequency of the maximum sea states were simulated with rather high accuracy.

The present surface wave models have mainly evolved from the WAM model by improvements in the parameterization of three basic source functions $S_{in}$, $S_{nl}$, and $S_{ds}$ on the right hand side of (5.51). The models are reasonably accurate for deep water (tens of centimeters to 0.5 m in significant wave height), though often much less accurate for the very low-frequency swell. The reliability of the results of these models depends critically on the accuracy of the wind field since wave memory is rather brief.

As indicated above, the total stress at the sea surface is the sum of a turbulent part and a wave-induced part: $\tau = \tau_t + \tau_w$. Theoretical and laboratory studies indicate that large amplitude surface waves may significantly modify the momentum flux at the air-sea interface, i.e., $\tau_w$ may be a considerable fraction of the total stress. Janssen (1989) developed a quasi-linear theory of wind-wave generation based on Miles (1957) theory that allowed coupling between wind and waves. He demonstrated that coupling with ocean waves depends sensitively on the age of the wind-generated waves, which is measured by the ratio $c_p/U_w$. The principle result of Janssen's theory is that for young wind waves ($c_p/U_w < 5-10$), which have a large steepness, momentum transfer from air to ocean increases considerably relative to a fully developed wave field $c_p/U_w > 25$. Jensen (1991) studied the interaction of wind and waves by coupling the WAM model with a simple surface-layer model that has a drag that depends on both wind speed and the rate of change of the wave-induced stress. Considerable sea-state dependence of the surface stress was found. Janssen (1992) used the quasi-linear theory of wind-wave generation of Janssen (1989) to modify the Charnock relation for the roughness by including a sea-state dependence. Comparisons between observed and modeled surface stress indicated that the sea state-dependent roughness length gave better agreement with observations than the traditional Charnock relation.

The effect of a wave-dependent drag coefficient on the generation of storm surges (see Chapter 5.7) was investigated by Mastenbroek et al. (1993). The momentum induced stress was calculated using Janssen (1991) theory and the sea state was calculated according to the WAM wave model. Mastenbroek et al. compared the results of the coupled wave-storm surge model with a model using only a standard wind dependent stress relation for three storms in the North Sea. The calculations with the wave-dependent drag gave a significant improvement (about 20%) in surge prediction. The most improvement was found in the case of a fast-moving storm, where the air-sea interaction was dominated by young waves.
Another important effect that must be taken into account for air-sea exchange in tropical cyclones is wave breaking. Under storm conditions, the short waves (lengths less than 1 m) are strongly involved in the breaking process. The breaking event (spilling breaker) is usually on the downwind slope of steep waves just beyond the sharpened crest. Because wave breaking occurs over a full spectrum of wave components, measurements and analysis are very difficult for real conditions. Banner (1990) studied the air flow above breaking monochromatic (single wavelength) waves in laboratory conditions. A large increase (typically 100%) of the total wind stress occurred above breaking waves, mainly owing to the increase of wave-coherent momentum flux (form drag) associated with actively breaking waves. The form drag arises from the correlation of the pressure field with the wave slope. Maat and Makin (1992) used a nonlinear numerical model to study the air flow above breaking waves. The breaking event was parameterized by enhancing the local roughness and changing the wave slope. Both mechanisms influenced the form drag and the total wind stress, which confirmed the conclusion of Banner (1990) on the drastic increase of form drag and, as result, of the total wind stress above steep breaking waves.

In summary, the studies discussed above provide support for the claim that large amplitude wind waves may play an important role in the air-ocean interaction. Strong coupling between sea state, surface winds, and ocean currents may occur during storms. However, a significant gap exists in our understanding of these effects for tropical cyclone conditions. A better knowledge of the dynamics of the sea surface may result in improved estimates of momentum, heat, and moisture fluxes, which play a key role in the evolution of a tropical cyclone. The improved knowledge will also be beneficial for storm surge prediction and calculating the ocean response to hurricane forcing.

5.7 STORM SURGES

A tropical cyclone making landfall may produce a storm surge, which is the abnormally high sea-level elevation. Combined with astronomical tides, peak water levels are generally sustained over a period of several hours depending on tropical cyclone size and translation speed. The definition of a surge does not include shorter term phenomena, such as surface waves, breakers, or spray. Typical increases of sea level associated with surges are several meters, but extreme surges of up to 10 m can occur. The spatial distribution and amplitude of the storm surge depend in complicated way on the bottom topography as well as the size, intensity, direction and speed of movement of the tropical cyclone.

Storm surges cause most of the damage associated with tropical cyclones in coastal areas. Certain low-lying areas adjacent to shallow seas are particularly vulnerable to surges. Historical events of the terrible devastation over coastal flood plains include the storm surge that flooded the island of Galveston, Texas, in 1900 and was responsible for 6,000 deaths. In 1969, Hurricane Camille generated a surge along the Mississippi Gulf Coast that peaked at more than 7 m above the mean sea level. This surge caused more than 100 deaths and close to one billion dollars in damage. In November 1970, a storm surge of an estimated maximum level of 9 m at Chittagong in Bangladesh cost 350,000 lives. Another storm-surge disaster in which about 140,000 people died was generated by the cyclone that struck Bangladesh in April 1991.

Increases in coastal population result in surge scenarios that may potentially be far worse than the disasters mentioned above. The potential severity of these disasters requires special forecasting techniques for estimating storm surge levels. The accurate surge predictions are essential for real-time warnings and for the design of coastal protective works that will reduce economic loss and human suffering. In an effort to minimize storm-surge devastation in future tropical cyclones, governments of many countries are developing regional evacuation plans. These plans require accurate estimates of the possible area/extent of storm-surge flooding. Various forecasting methods for making these estimates will be discussed in this section.
5.7.1 Statistical and empirical methods

These methods are based on analyses of the surge history that utilize tide and high water records for the area of interest. Sufficient data must be available so that the historical records can be considered representative of conditions to be expected in the future. In some cases, surge records from other similar locations can be used. It is important to note that data describing water, sea bed, and ground elevations along the shore must be referred to the same datum. Several different datum planes are in common use. For instance, in the U.S. these are: the National Geodetic Vertical Datum (NGVD), Mean Low Water, Mean Sea Level, and Mean High Water. Differences between the local mean sea level datums and NGVD, or other datums at various locations along the U.S. and Canadian coasts, are described by conversion factors or constants available from the U.S. National Ocean Survey.

Statistical analyses of surges typically involve various parameters such as: historical water levels, astronomical tides, storm strength and movement factors, strike probability, joint probability, and confidence limits. These terms are defined below.

**Historical water level** observations consist of tide and staff gauge records and high water marks. The tide and gauge data are usually contaminated during the tropical storm passage by high frequency wind-wave "noise" that must be filtered out. The usual procedure is to use either an interpolation technique or visual averaging, which introduces assumptions and subjective judgments that produce results that can never be used with absolute certainty. When gauge records at locations other than the area of interest are used, a datum adjustment relationship for the nongauge location may be required. This relationship is established by interpolating simultaneous records from more than one gauge as a function of coastal location. Unfortunately, tide and staff gauges are frequently inoperative during periods close to peak surge levels, which is a major impediment to statistical analyses. In these cases, compilation and assessment can be made of all available high water marks, especially those found inside structures that fully attenuate the short-term phenomena such as wind waves and spray. Such surge surveys are particularly important for comparing severe historical events with surge estimates when gauges have malfunctioned. However, water marks should be carefully checked and should not be accepted without verification. For example, high water marks based upon debris may represent short-term wave action and not longer period surge extremes. Care should also be taken in establishing a common datum for both high water marks and gauge records. Analyses of tide and staff gauge records, as supplemented by high water marks, are often required to estimate both astronomical and actual water levels.

The state of the astronomical tide is a very important factor in storm surge analyses. In the U.S., predicted astronomical tide levels that are published annually by the National Ocean Survey of NOAA include the heights and times of the high and low astronomical tide at certain reference stations. However, actual levels in the absence of a storm surge can vary from predicted astronomical levels and consideration of such tide level uncertainties should be made in assessing both historical and future conditions.

**Storm strength and movement factors** are usually defined by the following parameters: minimum central pressure of the storm, radius of maximum winds, translation speed and direction. If these parameters are considered random and independent, their coincident occurrence may be treated by statistical analyses. That is, the probability of a storm containing specific values of each parameter is the product of the probability of each value. Another major factor in the surge level at a specific location is the storm track with respect to the coast. An appraisal of how often storms pass the location (strike probability) and their approach directions are therefore required. Joint probability distributions of specific storm parameters and strike probability provide the statistical basis for assigning a probability of occurrence to the temporal and spatial characteristics of a surge. Difficulties in applying the statistical method for forecasting of infrequent events such as storm surges necessitate consideration of confidence limits, which provide a measure of the uncertainty.
of the surge level at a selected exceedence probability. Confidence limits narrow when
periods of record become longer. Thus, predictions of surge level or frequency should be
accompanied by explicit indications of the level of uncertainty in the prognosis.

5.7.2 Surge modelling

Although statistical methods are useful in surge forecasting, these methods are
sometimes impractical due to the lack of historical data. Numerical models provide an
alternate source of information concerning the range of expected peak surge heights in the
tropical cyclone warning area.

Several numerical models are currently being used to estimate hurricane-produced
surges. The hydrodynamic equations in most of these models are the momentum and
continuity equations depth-integrated from the sea surface at \( z = \zeta (x,y,t) \) to the bottom \( z = -H(x,y) \). The primary simplifications in the equations of motions include ignoring the
baroclinic effects (ocean stratification) and the assumption of depth-independence of the
storm-induced currents. In Cartesian coordinates with \( x \) increasing eastward, \( y \) increasing
northward, and \( z \) increasing vertically upward from an origin at the undisturbed sea surface,
the hydrodynamic equations of the surge models are

\[
\frac{\partial u_h}{\partial t} + \frac{\partial u_h u_h}{\partial x} + \frac{\partial v_h u_h}{\partial y} - f v_h = -g h \frac{\partial \zeta}{\partial x} - \frac{1}{\rho} h \frac{\partial p_a}{\partial x} + \frac{\tau_x^s - \tau_x^b}{\rho},
\]

\[
\frac{\partial v_h}{\partial t} + \frac{\partial u_h v_h}{\partial x} + \frac{\partial v_h v_h}{\partial y} + f u_h = -g h \frac{\partial \zeta}{\partial y} - \frac{1}{\rho} h \frac{\partial p_a}{\partial y} + \frac{\tau_y^s - \tau_y^b}{\rho},
\]

\[
\frac{\partial \zeta}{\partial t} + \frac{\partial u_h}{\partial x} + \frac{\partial v_h}{\partial y} = 0.
\]

In these equations, \( h = H + \zeta \); \( u \) and \( v \) are the components of the depth-averaged currents;
\( p_a \) is sea-level atmospheric pressure; and \( (\tau^s_x, \tau^s_y) \) and \( (\tau^b_x, \tau^b_y) \) are surface wind
stress and bottom frictional stress components, respectively.

Equations (5.52) - (5.54) are prognostic, and they may be used to make forecasts or
hindcasts of the surge by specifying various storm, geometry, and water level conditions.
They address two-dimensional, integrated currents since the main concern is with the surge
(sea-surface elevation) prediction. When computations of the storm-induced currents are
required (e.g., for design of offshore structures), a three-dimensional model must generally
be used. In contrast to the "full" models that predict the surge and three-dimensional
currents, two-dimensional models are much more economical and can easily be run on
personal computers. Surge predictions made by these models frequently give similar
accuracy as more sophisticated three-dimensional models.

Models applied for surge forecasting can be linear or nonlinear in various degrees.
Nonlinear models based on the full system of equations (5.52) - (5.54) include terms that
are numerically complicated. Nonlinear advective terms in the momentum equations
(5.52) and (5.53) are frequently neglected. However, the nonlinear water-height
perturbation terms in the continuity equation (5.54) are retained. Linear models are
simpler (and more economical) to use and the results are easier to interpret. Although
linear models are generally capable of reproducing the major characteristics of surges,
comprehensive tests with nonlinear models should be performed to evaluate likely errors in
the predicted elevations.

Important considerations for any numerical surge model include coastal
configuration, bathymetry, topography, size of computational domain and model
resolution. Bathymetry refers to the ocean depth in the coastal area, or on the continental
shelf. The spatial distribution of bottom depths is one of the most critical factors in surge generation. Higher surges are generated in regions where the continental shelf is wide and shallow because the convergence is enhanced. Subgrid-scale water features such as cuts, chokes, sills, and channels can generally affect the height of the storm surge and should be incorporated into a surge model. The topography of coastal areas is an essential factor in modelling the distribution of inland flooding induced by the surge. Topography here refers not only to the land elevations, but also to the surface cover and structures on the land (vegetation cover, dune heights, buildings, etc). Surface roughness is important in dissipating the energy of the surge and in slowing movement of water inland. Vertical obstructions such as levees, roads, spoil banks, etc. may also be parameterized. Another important factor in the surge is the shape of the coastline. A concave coast favors convergence, and thus higher surges. The coastal shape also significantly influences the alongshore coastally-trapped wave propagation resulted from hurricane forcing. Finally, the size of the computational domain and grid resolution must be based on economic considerations as well as the physical conditions.

Accurate solutions of equations (5.52)-(5.54) require the proper specification of initial and boundary conditions. The initial conditions specify the water level before the storm arrival and must be carefully considered due to possible nonlinear, finite amplitude effects. In the absence of a surge, water levels may vary due to several causes. Surges can be catastrophic or cause no damage depending on the stage of the astronomical tide. Some cases of this effect based on historical observations in China are described by Zhang et al. (1993). Another level variation to be considered in the model is the sea-level anomalies. In general, the occurrence of surges during periods of high ambient water levels unrelated to astronomical tide may cause severe consequences. Unfortunately, the present models do not include these effects. Even the interaction of the storm surge and astronomical tide has yet to be treated dynamically in most surge prediction models. Instead, surge forecasters generally superpose a tide elevation onto the computed surge. However, the nonlinear surge-tide coupling might be an important factor to consider for large amplitudes of tides and surges. For example, astronomical tidal ranges along the U.S. Gulf of Mexico coast are generally less than 1 m, but increase from 2 m along the southern Atlantic coast to 4 m near Boston, and to more than 7 m along the Maine coast.

An appropriate specification of coastal boundary conditions is another important factor in successful surge prediction. Three basic types of the coastal boundary conditions have been applied to surge models: (a) fixed vertical wall of finite height; (b) fixed vertical wall of infinite height; and (c) moving boundaries that allow flooding and inundation. Because the coastline geometry plays an important role in surges, special care is required to design a finite-difference approximation that has the maximum possible accuracy. Since the coastline in the most surge models is approximated by straight line segments in either the x or y directions, a fine grid resolution is highly desirable. In some models developed for the Bay of Bengal (e.g., Johns et al. 1981, Dube et al. 1985), a coastline-following curvilinear system of coordinates is used. Comparisons by Johns et al. (1982) of both types of boundary treatments show that the step-wise boundary treatment generally tends to underestimate the surge response.

The lateral conditions are the boundary conditions on the open-ocean ends of the basin. They can be closed, transport specified, or radiation type. A common goal is to construct the boundary conditions that maintain smooth fields near the boundary without causing a noticeable erroneous impact on the interior domain solutions. If a storm is landfalling, the lateral boundary is generally not as critical as when the storm is moving along the shore. One possible technique is a movable grid in which the computational domain is shifted to the next grid as soon as the storm approaches the boundary. Another approach is to use overlapping coarse and fine meshes in which the model solutions on the coarse mesh are used to supply the boundary values for the fine-mesh model.

Because the surge is a forced wave, a realistic specification of hurricane forcing is critical for successful forecasting. As follows from (5.52)-(5.54), the two forcings to be
specified are the wind stress and atmospheric sea-level pressure. Sensitivity analyses indicate that the major contribution to the storm surge comes from the wind stress. As a result, the atmospheric pressure forcing is either neglected in some models or is added at the end of the integration in the form of an inverted barometer effect (~ 1 cm per millibar drop of pressure).

Specification of the wind stress requires a large number of near-surface observations that are notoriously hard to obtain for hurricane conditions. Thus, the wind stress must be parameterized for operational surge predictions. This specification is the main discrepancy among surge models. Bulk wind stress relationships used in many models have not been universally accepted primarily because of theoretical considerations and lack of velocity and sea surface information at high wind speeds. The main question related to the form of the bulk relationship is the value of the drag coefficient, which is assumed to be constant in some models or be variable with wind speed. The Charnock (1955) relation for the roughness length, which has a roughly linear relation between the drag coefficient and the wind is commonly used in storm surge modelling. As indicated in Chapter 5.6, the wind stress may depend not only on the wind speed but also on the sea state. Mastenbroek et al. (1993) have recently reported a first attempt to force a barotropic storm surge model with a wind stress that depends explicitly on the ocean waves. The WAM wave model was used to calculate the waves and the wave dependence of the drag coefficient. The theory of Janssen (1991) was used to estimate the effects of waves on the boundary layer. These coupled wave and storm surge models were tested for three storm surges in the North Sea.

Even if the parameterization of the wind stress was correct, the problem is to determine the surface winds in a tropical cyclone. Because adequate wind data are frequently lacking, the practical approach is to use a storm model based on available meteorological parameters such as minimum sea-level pressure, maximum surface winds, and radius of maximum winds. For operational forecasting, simple empirical storm models are usually used based on these parameters. However, no general consensus exists as to the best empirical model. Because each surge model has a different specification of surface winds, it is difficult to compare models. Storm surge models are commonly "tuned" to the observations from a number of post-storm analyses by varying the storm parameters and/or the drag coefficient.

Another subgrid-scale process that needs to be parameterized in any surge model is the bottom stress \((\tau_x^b, \tau_y^b)\), which may have a significant effect on surges in areas of shallow depth. For the momentum equations (5.52)-(5.54), the parameterization of the bottom stress must be in terms of the depth-averaged current. Again, no consensus exists, mainly due to the lack of observations and a firm theoretical basis. The bottom stress is usually represented by the quadratic relationship

\[
(\tau_x^b, \tau_y^b) = k \rho (u^2 + v^2)^{1/2} (u, v),
\]

where \(k\) is an empirical friction coefficient that is often taken to be a constant. The empirical formula (5.55) is clearly crude since the coefficient \(k\) must generally be a function of space and time. Therefore, misrepresentation of the bottom stress may lead to substantial errors in surge prediction. Johns (1981) compared three-dimensional, multi-level model calculations in which bottom friction was calculated using a turbulent closure scheme with the predictions based upon the model set (5.52)-(5.54) with a constant coefficient \(k = 4.63 \times 10^{-3}\). He found that the differences in surges ranged from 23% to 80% depending on location in the very shallow regions.

The drag and bottom friction coefficients in the numerical models may be viewed either as a representation of a physical process, or as a convenient way of calibrating the model against experimental or historical data. In view of the uncertainties in the parameterizations discussed above, these coefficients may be considered as model "knob turners" that provide a convenient way of calibrating a model against observed data.
Interpreted in this manner, the coefficients are indicative of the surge and hurricane forcing model inaccuracies and, therefore, will vary among different models. Consequently, two necessary aspects in developing a surge prediction model are the calibration (parameter evaluation) and the verification (reliability and range of applicability). During the calibration process, parameters are adjusted until the model outcome satisfactorily agrees with experimental or historical observations. Error criteria are usually chosen that minimize the differences between observed and computed maximum surge levels at selected sites. Since most of the storm-surge models are site-dependent, the process must be repeated for each new application of the model. Calibration and verification should be executed on different data sets. In practice, this may be difficult to accomplish because of the limited storm surge data sets. Even if historical surge data sets exist, the quality of the data with respect to spatial and temporal distributions may be poor.

Finally, other potentially important physical processes that might be included in surge prediction models are wind waves, rainfall, and river flows. Breaking waves at the shore usually force water inshore and cause the so-called "wave set-up." Wave set-up will contribute to the total water level and should be considered in evaluating model results against historical data that may or may not include this effect. Along most of the U.S. coastline, larger waves generally break at substantial distances from the coast, which results in a reduced wave set-up at the coast. If the shelf is narrow, larger waves break near the coast and, therefore, a significant wave set-up potentially exists. Wind wave effects are also important at the coastlines since they contribute to the structural and erosion damage.

Although tropical cyclones usually produce large rainfall amounts, this is a relatively small surcharge of water compared to the surge height. However, heavy hurricane rainfall often causes floods in streams entering an estuary. If the high streamflows are nearly concurrent with high tide levels, the storm surge effects will be amplified within the tidal reach of the stream. That is, the discharge of the stream will also add to the volume of water in the main body of the estuary.

5.7.3 Surge predictions

Significant progress has been made during the past two decades in numerical predictions of storm surges. Surge prediction systems, which provide warnings of impending flood conditions, have been designed and are now operational in many countries that suffer from frequent landfalls of tropical cyclones. The systems are usually operated by the national weather forecasting centers. The data and warning are distributed to numerous users, including the offshore industry, navigation authorities, flood defense authorities, coast guard, and navy.

In the U.S., the SLOSH (Sea, Lake, and Overland Surges for Hurricanes) storm-surge model has been developed by the National Weather Service (Jarvinen and Lawrence 1985). SLOSH is an extension of a prior model called SPLASH (Special Program to List Amplitudes of Surges from Hurricanes) developed by Jelesnianski (1972). Both SLOSH and SPLASH models are based on the linearized equations (5.52)-(5.54). Details of the model formulations are discussed by Jelesnianski (1965, 1966, 1967). In the SPLASH model, storm-surge heights are computed only over water and at the coastline (which is assumed to be an infinite vertical wall). In the SLOSH model, computations extend inland over the coastal flood plain. Bathymetric and topographic map data are used to assign a water depth or terrain height to each model grid point. These data are obtained from charts prepared by the National Ocean Survey and the U.S. Geological Survey, and also by on-site surveys. The model is forced by a time-varying horizontal distribution of wind stress and pressure gradient calculated by a method developed by Jelesnianski and Taylor (1973). Astronomical tides have not been incorporated into the SLOSH model. An evaluation of the SLOSH model for 523 observations of storm-surge height during 10 hurricanes (Jarvinen and Lawrence 1985) indicates that under ideal conditions in which the hurricane track and intensity are known, model surge-height errors are less than 0.6 m for 79% of the
observations and exceed 1.8 m for only 1% of the observations. An example of the SLOSH model performance for surge computation during Hurricane Hugo in 1989 is shown in Fig. 5.29. The computed surge values using a post-storm data analysis of the best track and storm parameters demonstrate a rather good agreement with observations.

The Australian Bureau of Meteorology Research Center (BMRC) has developed an efficient surge forecast system as part of the Australian Tropical Cyclone Workstation project (Hubbert et al. 1991). A full description of the surge model, which is based on the depth-integrated equations (5.52)-(5.54), is given by Hubbert et al. (1990). A split-explicit time integration scheme is applied on an Arakawa C-grid to solve numerically the model equations. Atmospheric forcing is derived either from a tropical storm prediction model or from an empirical model of Holland (1980). In the latter case, the surge model can be used in a stand-alone mode and requires only information on cyclone positions, central pressures, and radii of maximum winds. Hubbert et al. (1991) tested the model for four Australian tropical cyclones. When run at a resolution of 15 km, the model simulated quite accurately the sea-surface elevations and arrival times of the peak surge. Because the BMRC surge forecast system requires only a few minutes to run on a personal computer, multiple forecast scenarios may be made in real time. The model is now being used by the Australian Tropical Cyclone Warning Centers for operational forecasting.

A number of surge models have been developed and used for the Bay of Bengal. Das et al. (1974) used the linearized equations (5.52)-(5.54) to estimate the maximum surge height from basic storm parameters. Ghost (1977) adopted the SPLASH model to the east coast of India. A hierarchy of four models of increasing complexity has been developed in the Centre for Atmospheric Sciences, Indian Institute of Technology, to simulate storm surges in the Bay of Bengal (see review by Johns and Lighthill 1993). The first three models are based on the depth-averaged formulation (5.52)-(5.54) with an empirical representation of the bottom friction (5.55). They employ a non-orthogonal, coastline-following coordinate and apply radiation boundary conditions at all lateral boundaries.
The primary difference among the models is in specification of the coastal boundary conditions: fixed side-wall (Johns et al. 1981); or variable coastline (Johns et al. 1982). The third model (Johns et al. 1983) has increased cross-shelf resolution. The models have been tested with observations from the 1970 surge that affected the coastal region of the Indian state of Andra Pradesh. The wind stress was parameterized by the bulk formula, and the surface winds were specified by an empirical formula suggested by Jelesnianski (1965). The fourth model is fully three-dimensional (Johns et al. 1983). Interestingly, the simulations with depth-averaged models (with $k = 2.6 \times 10^{-3}$) have only about 9% differences in sea-surface elevations. All model solutions clearly demonstrate an alongshore coastally-trapped wave propagating along the east coast of India. This edge-wave response to tropical cyclone forcing is discussed by Johns and Lighthill (1993).

Several successful simulations of storm surges for India, Bangladesh, and Sri Lanka were reported by Dube (1993). These simulations were made with a prediction model based on the equations (5.52)-(5.54) adopted for the Bay of Bengal and the Arabian Sea. As for the BMRC surge model, this model can be run on a personal computer, which allows a surge forecast to be made in real time at little cost. More recently, a new storm surge prediction model for the northern Bay of Bengal was developed by Flather (1994). The model combines in a unified numerical framework the 1-D equations for narrow river channels, 2-D equations for the open sea, and an approach to modelling inundation. Whereas the models of the region discussed above ignore the complex network of linked channels that make up the Ganges Delta, Flather’s model includes the delta in realistic manner. Atmospheric forcing is derived from the empirical Holland (1980) model combined with storm data from JTWC. The model was successfully applied to simulate the disastrous storm surge caused by the Bangladesh tropical cyclone in April 1991. The hindcast of this event reproduced the storm surge and extent of flooding with a good skill.

Qin (1993) reported significant progress in surge modelling in China during the last ten years. Three- and two-dimensional models are currently used for analyses of physical processes related to surges and flooding. For example, considerable contributions due to nonlinear tide-surge interactions have been identified for some areas of the China coast. Comparisons with observations showed that a simple linear superposition of astronomical tide with a separately computed surge may lead to errors of 1-2 m in surge prediction (Zhang et al. 1993). It has also been found that bottom friction has a great influence on the storm surges in the Yellow and East China seas, especially if the tropical cyclones move parallel to the coastline. Numerical experiments indicated that use of different friction coefficients for various areas could give a better surge prediction than the use of a universal value.

To summarize the capability of many of the countries, the models in the surge forecasting systems typically are two-dimensional in the horizontal to account for all significant bathymetric and coastline features, and are depth-integrated in the vertical. The combination of meteorological and hydrological components is different from model to model. It is emphasized that the major differences among the models in terms of the storm parameterization need to be resolved. Any comparison of surge models must first establish the "best" characterization of meteorological input before consideration of hydrological factors, and ultimately surge predictions, can be made.

Several storm surge models appear to be capable of predicting accurately the water height (several tens of centimeters, generally) if accurate tropical cyclone data are provided. Therefore, great improvements in storm surge predictions would follow from improvements in forecasting the tropical cyclones.

5.7.4 Nomograms and surge atlases

Although some models appear to have considerable accuracy in these surge predictions, these cases have usually been made with a "perfect" knowledge of storm track, size, and intensity. Because of the extreme sensitivity of the storm surge to these characteristics of the tropical cyclone, accurate forecasts of storm surge depend critically on
accurate forecasts of these storm characteristics. For example, the surge affects about 150-200 km of coastline for a typical landfalling storm. However, the average error of 24-h tropical cyclone track forecasts is also about 200 km. If predicted tracks are used to forecast the storm surge, then the forecast location of the peak surge can be expected to have approximately the same average error as the track forecasts. Some examples of such errors are given by Jelesnianski (1993). Most forecasting offices do not have access to sufficient computer power to accomplish model computations with several tracks, storm intensities and sizes in a real-time situation, which would require many computer runs and time-consuming analyses of the outputs. Given the present state of storm-track forecasting accuracy, surge models have only limited utility for real-time operational predictions.

An alternate approach is to construct nomograms that show the quantitative effects of storm intensity, speed, and direction of motion on the storm surge. These nomograms may be constructed in advance by simulating storm surge effects along the coastal topography for many different model storms. Besides their obvious value in land-use planning, these nomograms may be useful for surge forecasting and for evacuation planning when tropical cyclones threaten. Nomograms calculated by the SPLASH model are used operationally for surge predictions in the East China Sea (Qin 1993).

Pre-computed nomograms are also used at the India Meteorological Department for operational surge predictions in the North Bay of Bengal. The nomograms have been constructed from modelling studies of a large number of bathymetries, storm parameters, and land approach angles using the SLASH model adapted by Ghosh (1977). By using these pre-computed nomograms, Mandal (1991) prepared the Probable Maximum Storm Surge for the Indian and Bangladesh coastlines (Fig. 5.30). According to this nomogram, the highest storm surge in the North Bay of Bengal (13.2 m) may be expected near the city of Hatia.

Using climatological data of tropical cyclones affecting a given region, an atlas of pre-computed surges can be prepared. To produce such an atlas, a family of idealized storms that are likely to affect the area is constructed based on tropical cyclone parameters such as track directions, intensities, and sizes. In the U.S., flood-prone areas are usually determined by using the SLOSH model and input parameters from 200-300 hypothetical hurricanes. Separate composite flood maps are produced for up to five levels of hurricane intensities (Jarvinen and Lawrence 1985).

Jelesnianski (1993) gives a detailed description of the methods for preparation and usage of the nomograms and surge atlases. Jelesnianski emphasizes the particular importance of a pre-computed map of maximum surge values at all vulnerable coastal locations from a family of cyclones, regardless of which cyclone is responsible. The map, which is called a Maximum Envelope of Waters (MEOW), provides information on the "worse case" scenario, and is especially valuable given the uncertainties in storm predictions. It is recommended that any atlas of pre-computed surges contain sets of MEOW, which the U.S. National Hurricane Center has found useful to forecast storm surges.

In summary, significant progress has been made in developing surge prediction models and research is continuing in many countries with the common goal of improving the capability of their surge forecasting systems. These include predictions of water levels, currents, and waves, with an increasing emphasis on the depth-dependent density and velocity components. Future areas of improvements will involve interactions among the different fields, i.e., wave-current interactions; assimilation of observations into the analyses from buoys, satellites, and tide stations. There also is a continuing interest in the use of real-time data for updating the predictions of surge models by assimilating observations to provide better analyses of the present situation. However, little of the potential capability and power of data assimilation is currently operational, so opportunities of future improvements exist.
Fig. 5.30 Probable maximum storm surge (m) for the Indian and Bangladesh coastlines constructed by using pre-computed nomograms (Mandal 1991).

REFERENCES


